Economic Growth and the Design of Search Engines

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A little bit of anticipation

- Suppose the internet becomes a major device for allocating goods to consumers
- Then a good chunk of GDP is going to be intermediated via the web
- *Search engines* will play a crucial role in the allocation of goods to consumers
- A natural question is then: How does the design of search engines affect GDP growth?

This paper

- Develops a model where a very large number of goods is available
- Consumers use search engines to locate the « best » goods
- The expected visibility of a new good affect the profitability of innovation
- This in turn potentially affects the growth rate

Two effects

- Visibility: a greater score increases market size and therefore profits
- Selection: greater visibility of future competing goods makes a product obsolete earlier, which reduces the profits from innovation.
- For simple specifications the two effects cancel out→SE only has a level effects

Two models yield non neutralities

- The popularity model: visibility gradually builds up over time. This introduces a discount effect in the visibility effect that is absent from the selection effect.
- The advertising model: firms improve their visibility by using advertising; but advertising uses resources and resources left for R and D are reduced.

A basic model

The economy is populated by L individuals, each endowed with one unit of labor and an equal claim on profits. At each date t there is a continuum of goods available for consumption. The total mass of goods is N_t. Goods differ by their quality q. The quality distribution is invariant over time and given by the c.d.f.

$$F(q) = 1 - \mathrm{e}^{-\lambda q}; \tag{1}$$

the corresponding density is therefore

$$f(q) = \lambda \mathrm{e}^{-\lambda q}. \tag{2}$$

Innovation

- Innovators introduce new goods
- The quality of a new good is randomly drawn from the distribution $f()$
- 1 unit of labor = γN_t new blueprints
- Production costs are fixed and equal to $c=1$
- Prices are fixed and equal to $p=1$ (No competitor below p)
- This needs to be fixed…

Preferences

For each available good, consumers can consume either one or zero units. They get a utility flow equal to the quality of the good q. Their total flow of utility at t is

$$U_t = \int_0^{+\infty} \omega(q) g_t(q) \mathrm{d}q,$$

where $g_t(.)$ is the local mass of goods of quality q being consumed, which can never exceed $N_t f(q)$, and $\omega()$ is an increasing function.

Selection

At date t we denote by C_t the total quantity of goods being consumed. Each representative individual then consumes C_t/L goods. Since one unit of each good is consumed, and all goods have the same price while higher quality goods generate higher utility, consumers will consume one unit of all the available goods above some quality threshold q_t^* which satisfies

$$\frac{C_t}{L} = N_t(1 - F(q_t^*)). \tag{3}$$

This yields a utility flow equal to

$$U_t = N_t \int_{q_t^*}^{+\infty} \omega(q) f(q) \mathrm{d}q. \tag{4}$$

Obsolescence

In what follows I confine the analysis to balanced growth path where N_t grows at a constant rate and therefore C_t is constant through time. As long as the growth rate of N is strictly positive, it must be that q_t^* grows with time and tends to infinity so that the RHS of (3) stays constant. Consequently, each good of quality q eventually becomes obsolete at a critical date $T(q)$ such that $q_{T(q)}^* = q$. After this critical date consumers no longer consume that good as they can spend all their money on higher quality goods.

Equilibrium

- Labor market equilibrium:

$$\dot{N}_t = \gamma N_t (L - C_t).$$

- Value of an innovation:

$$V_t = \left(\int_{q_t^*}^{+\infty} f(q) \int_t^{T(q)} \mathrm{e}^{-r(u-t)} \mathrm{d}u \right) (p-1)L.$$

- R and D equilibrium:

$$V_t = \frac{1}{\gamma N_t}.$$

Computing a BGP (1)

We are now in a position to use the convenient exponential distribution to further characterize the BGP. Let g be the constant growth rate of N, and $\bar{C}$ be the constant value of C. Substituting (1) into (3) yields

$$\frac{\bar{C}}{L} = N_0 e^{gt} e^{-\lambda q_t^*}, \tag{8}$$

i.e.

$$q_t^* = a + gt/\lambda,$$

with

$$a = \frac{1}{\lambda} \ln\left(\frac{N_0 L}{\bar{C}}\right).$$

Computing a BGP (2)

Next, we have that $T(q) = \frac{\lambda}{g}(q - a)$. Substituting this and (2) into (6), computing the integrals and rearranging, yields

$$V_t = \frac{(p-1)L}{r} \mathrm{e}^{-\lambda q_t^*} \left[\frac{r}{r+g} \right],$$

which, after substituting (8) is simply equal to

$$V_t = \frac{p-1}{r+g} \frac{\bar{C}}{N_t}. \tag{9}$$

The equilibrium growth rate

$$g = \frac{(p-1)\,\gamma L - r}{p}.$$

Constant hit probability

- We consider the most simplistic search engine
- People locate goods with constant probability ρ
- ρ is the same across consumers and goods
- The set of goods that get a hit is uncorrelated across consumers

Consumers are less selective…

$$\frac{C_t}{L} = \rho N_t(1 - F(q_t^*)). \tag{11}$$

With our exponential distribution, that is equivalent to

$$\frac{\bar{C}}{L} = \rho N_0 \mathrm{e}^{gt} \mathrm{e}^{-\lambda q_t^*}, \tag{12}$$

…but firms have a lower chance to be located.

$$V_t = \left(\int_{q_t^*}^{+\infty} f(q) \int_t^{T(q)} \mathrm{e}^{-r(u-t)} \mathrm{d}u \right) (p-1)\rho L.$$

As a result, growth is unchanged

$$q_t^* = a' + gt/\lambda,$$

with

$$a' = \frac{1}{\lambda} \ln\left(\frac{\rho N_0 L}{\bar{C}}\right).$$

Thus $T(q) = (q - a')\lambda/g$. The same computations as above now yield

$$\begin{aligned} V_t &= e^{-\lambda q_t^*}\left[\frac{\rho(p-1)L}{r+g}\right] \\ &= \frac{p-1}{r+g}\frac{\bar{C}}{N_t}. \end{aligned}$$

Constant # of hits:

$$\frac{C_t}{L} = K(1 - F(q_t^*)). \tag{14}$$

As the number of goods grows relative to the number of hits, the market share of each goods falls with time as its chances of being located fall. The probability of being located at date t is K/N_t. Therefore, the PDV of introducing a new good at t is now given by

$$V_t = \left(\int_{q_t^*}^{+\infty} f(q) \int_t^{T(q)} \frac{K}{N_u} \mathrm{e}^{-r(u-t)} \mathrm{d}u \right) (p-1)L. \tag{15}$$

In a balanced growth path, C_t is constant, and so is q_t^*: The obsolescence process is shut down and $T(q) = +\infty$. Integrating (15), we find that

$$V_t = \frac{K(p-1)L\mathrm{e}^{-\lambda q_t^*}}{(r+g)N_t}.$$

Using the same steps as above we can show again that the growth rate is independent of K and still given by (10).

Popularity

- I now assume the SE rewards quality (*q*)
- But it can only do so by observing popularity going up over timer
- And it takes time for popularity to build up
- Gives an edge to existing goods over new goods
- But I know that if my product is good I will eventually get more customers

The trade-off:

$$\pi_t(q, s) = \rho(1 - k\mathrm{e}^{-\varepsilon(q-q_t^*)})(1 - \mathrm{e}^{-\alpha(t-s)}).$$

The structure of the SE:

- ρ is a measure of the overall efficiency of the search engine. An increase in ρ increases the number of hits proportionally for all goods. For convenience I assume $\rho < 1$.
- α is the speed of convergence to the target level of hits, which reflects the fact that it takes time to build popularity. The higher α, the lower the popularity advantage of older goods over newer goods.
- ε is the sensitivity of the target level of hits to quality. The higher ε, the greater the number of hits of the higher quality sites over the lower quality sites.
- k is a weight which captures the importance of quality; it will be treated as a fixed parameter.

The speed/accuracy trade-off

$$\alpha + b\varepsilon \leq \delta.$$

The average score of a good:

$$
\begin{aligned}
\pi_t(q) &= \int_{-\infty}^{t} g e^{g(s-t)} \pi_t(q,s) \mathrm{d}s \\
&= \frac{\alpha\rho}{g+\alpha}\left(1 - k e^{-\varepsilon(q-q_t^*)}\right).
\end{aligned}
$$

Quality threshold determination

$$
\begin{aligned}
\frac{C_t}{L} &= N_t \int_{q_t^*}^{+\infty} \pi_t(q) f(q) \mathrm{d}q \\
&= N_t \mathrm{e}^{-\lambda q_t^*} \frac{\alpha\rho}{g+\alpha} \left(1 - \frac{\lambda k}{\lambda + \varepsilon}\right).
\end{aligned}
$$

The value of an innovation

$$R_t(q) = (p-1)L\rho \int_t^{T(q)} \mathrm{e}^{-r(u-t)}(1 - k\mathrm{e}^{-\varepsilon(q-q_u^*)})(1 - \mathrm{e}^{-\alpha(u-t)})\mathrm{d}u.$$

$$V_t = \int_{q_t^*}^{+\infty} R_t(q) f(q) \mathrm{d}q$$

Equilibrium growth rate:

$$(p-1)(\gamma L - g) = \frac{(r+g)(r+g+\alpha)}{g+\alpha}.$$

The irrelevance of ε

- ε increases my sales if my good is of higher quality relative to others
- This upward risk increases my expected profit per unit of time
- But it allows consumers to become more choosy, thus raising q^*.
- Obsolescence occurs earlier.
- The two effects on growth cancel

The relevance of α

- A higher α reduces the advantage of older firms
- Innovators are more visible earlier
- Consumers are again more choosy
- The former effect brings profits forward in time ➔ reinforced by discounting
- The latter only depends on a more favorable cross-sectional distribution of q

Implications:

- The growth rate is increasing in α.
- For growth purposes, quick visibility of new products is more important than a search engine elastic to quality
- However, consumers only care about a product being high quality rather than new
- Unclear that « equilibrium » SE would have features that are best for growth

The advertising model

- Firms can invest resources to improve their score
- These resources cannot be allocated to alternative uses
- My score increases the pace of obsolescence by allowing consumers to see more goods and thus be pickier.

The score function:

$$\pi(\sigma) = 2\alpha\sigma^{0.5} + \rho.$$

Optimal advertising

$$\max_{\sigma} \pi(\sigma)(p-1)L\int_{s}^{T(q)} \mathrm{e}^{-r(u-s)}\mathrm{d}u - \sigma$$

The optimal σ is

$$\sigma(q,s) = \left(\alpha(p-1)L\frac{1-e^{-r(T(q)-s)}}{r}\right)^2.$$

Scoring:

$$\pi(q,s) = 2\alpha^2(p-1)L\frac{1-e^{-r(T(q)-s)}}{r} + \rho$$

$$\begin{aligned}
\pi_t(q) &= \int_{-\infty}^{t} g e^{g(s-t)}\pi(q,s)\mathrm{d}s \\
&= \frac{2\alpha^2(p-1)L}{r} + \rho - \frac{2\alpha^2(p-1)Lg}{r(g+r)}e^{-r(T(q)-t)}.
\end{aligned}$$

Scoring and quality:

- More profitable to invest in advertising if quality higher
- Thus total score goes up with quality
- Advertising introduces a link between the two, like popularity
- In principle, this is « good » for welfare

Selectivity

$$\frac{C_t}{L} = N_t \int_{q_t^*}^{+\infty} \pi_t(q) f(q) dq.$$

$$\frac{C_t}{L} = N_t \mathrm{e}^{-\lambda q_t^*} \left[\rho + 2\alpha^2 (p-1) L \frac{r+2g}{(r+g)^2} \right]$$

$$q_t^* = a''' + \frac{g}{\lambda} t.$$

$$a''' = \frac{1}{\lambda} \ln \left(\frac{N_0 L}{\bar{C}} \left(\rho + 2\alpha^2 (p-1) L \frac{r+2g}{(r+g)^2} \right) \right).$$

Revenues

$$R_t(q) = \left(\alpha(p-1)L\frac{1-e^{-r(T(q)-t)}}{r}\right)^2 + \rho(p-1)L\frac{1-e^{-r(T(q)-t)}}{r}.$$

$$\begin{aligned} V_t &= \int_{q_t^*}^{+\infty} R_t(q) f(q) \mathrm{d}q \\ &= \mathrm{e}^{-\lambda q_t^*} \frac{(p-1)L}{r+g}\left[\rho + 2\alpha^2(p-1)L\frac{1}{2r+g}\right]. \end{aligned}$$

Equilibrium conditions

In equilibrium, the growth rate is given by

$$g = \gamma(L - C_t - M_t), \tag{24}$$

where

$$M_t = \dot{N}_t \int_{q_t^*}^{+\infty} \sigma(q,t) f(q) dq \tag{25}$$

Growth determination

$$pg + r - \gamma L(p-1) = \eta H_1(g),$$

where

$$\eta = \frac{\alpha^2}{\rho}$$

and $H_1(g)$ is a function determined by

$$H_1(g) = -2(p-1)L\frac{r+2g}{r+g} + (p-1)^2 L\left[\frac{2\gamma L}{2r+g} - \frac{g}{r}\left(\frac{2r-g}{2r+g} + \frac{g+r}{r}\right)\right].$$

The result:

It can be shown straightforwardly that $H_1'(g) < 0$, therefore there exists a unique solution to (32).[6] Furthermore, the parameters of the search engine only enter through the ratio η, and it multiplies the RHS of (32). The equilibrium growth rate g^* increases (resp. falls) with η if and only if $H_1(g^*) > 0$ (resp. $H_1(g^*) < 0$). We can show[7] that $H_1(g^*) < 0$, therefore a search engine which is more elastic to the innovator's search input always reduces the long-run growth rate of the economy.

Why?

- Here α has a stronger (adverse) impact on selectivity than on profitability.
- This is because revenues are less sensitive to α than scores.
- Compounded by even more subtle effects…
- + direct resource drain of greater advertising.

Welfare

- This analysis suggests that sophisticated search engines do not beat random ones in terms of growth
- Things are different in terms of welfare: a higher score of high quality goods has a positive level effect on utility.

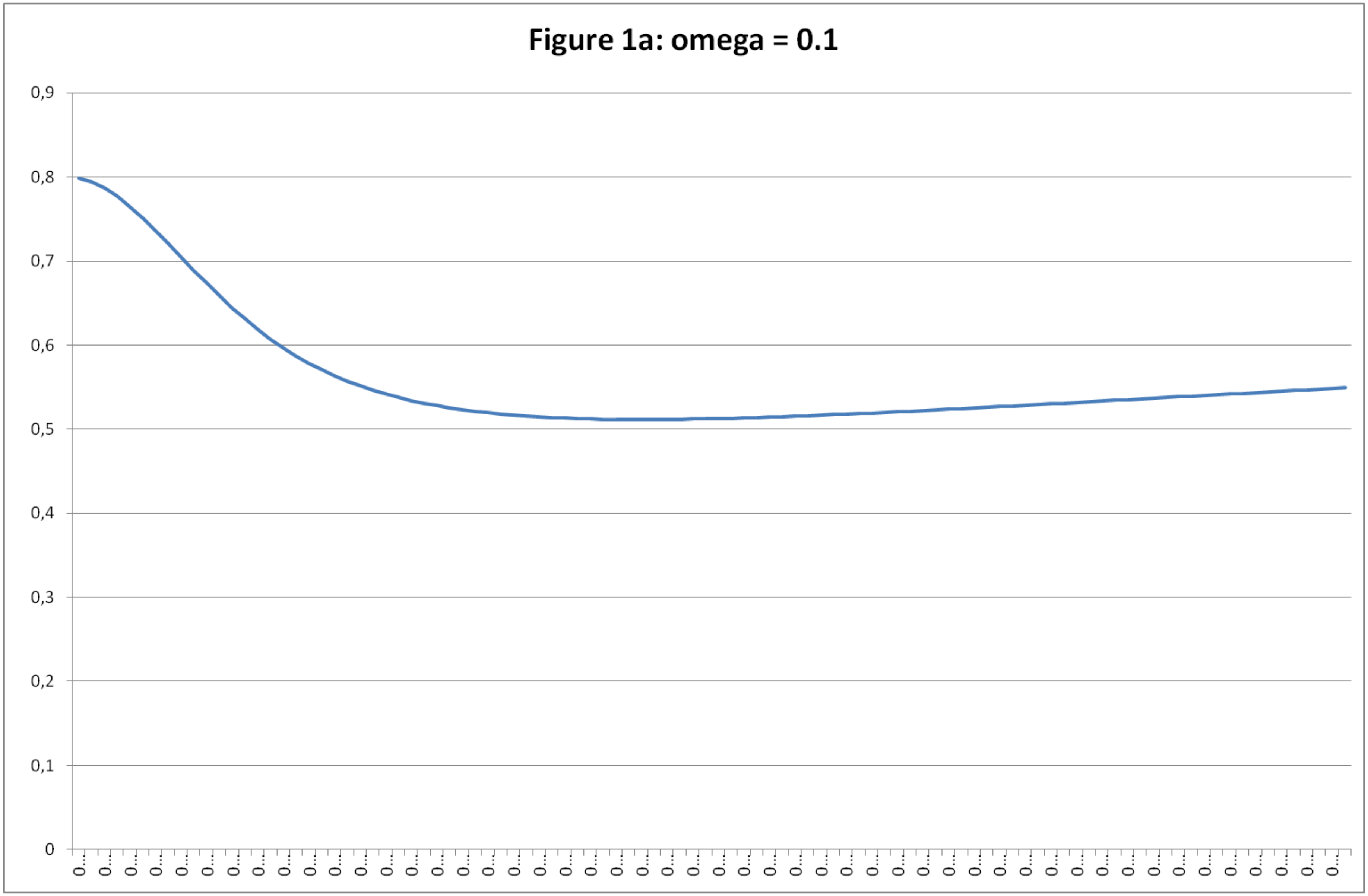
Figure 1a: omega = 0.1
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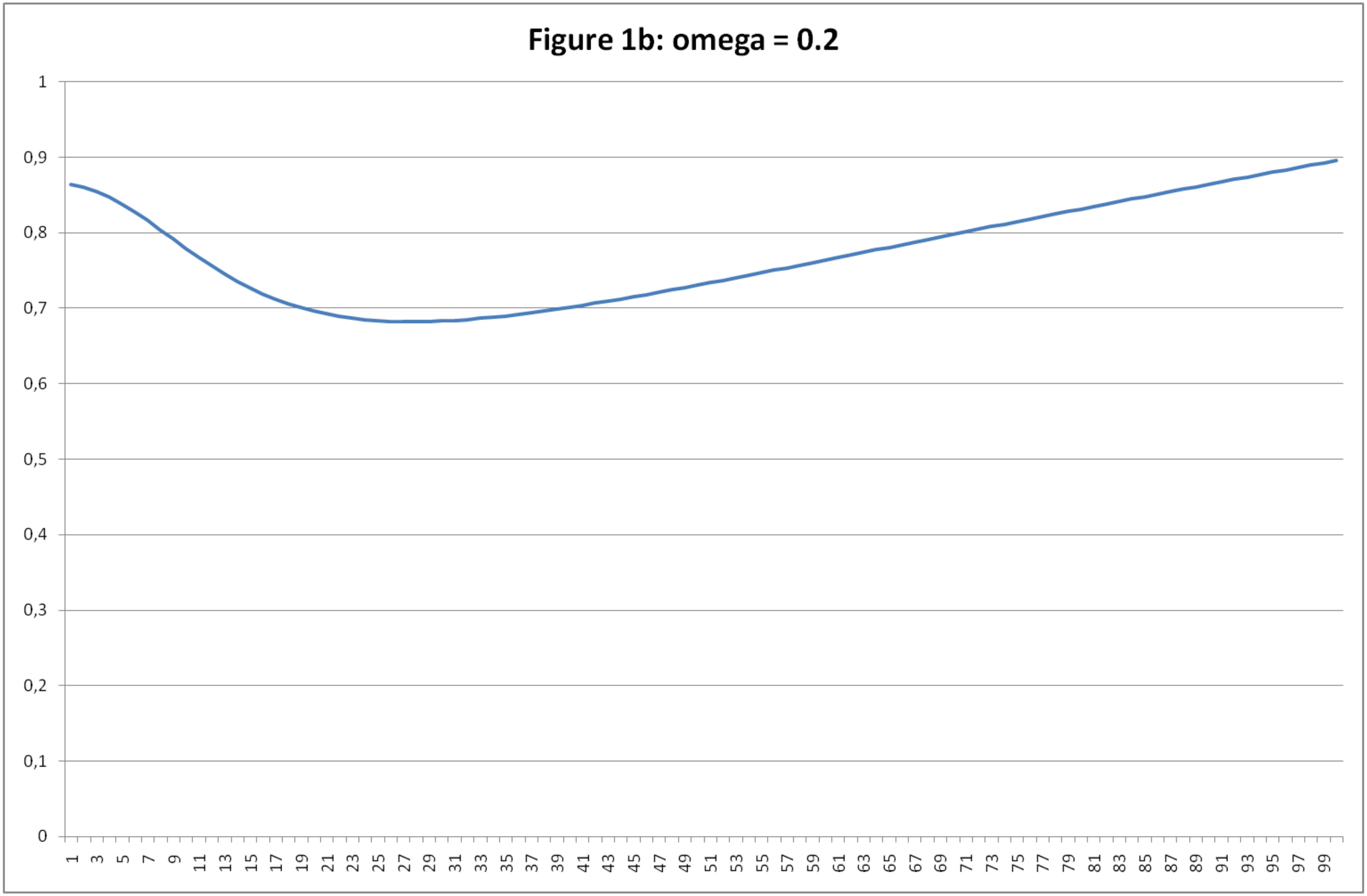

Figure 1b: omega = 0.2
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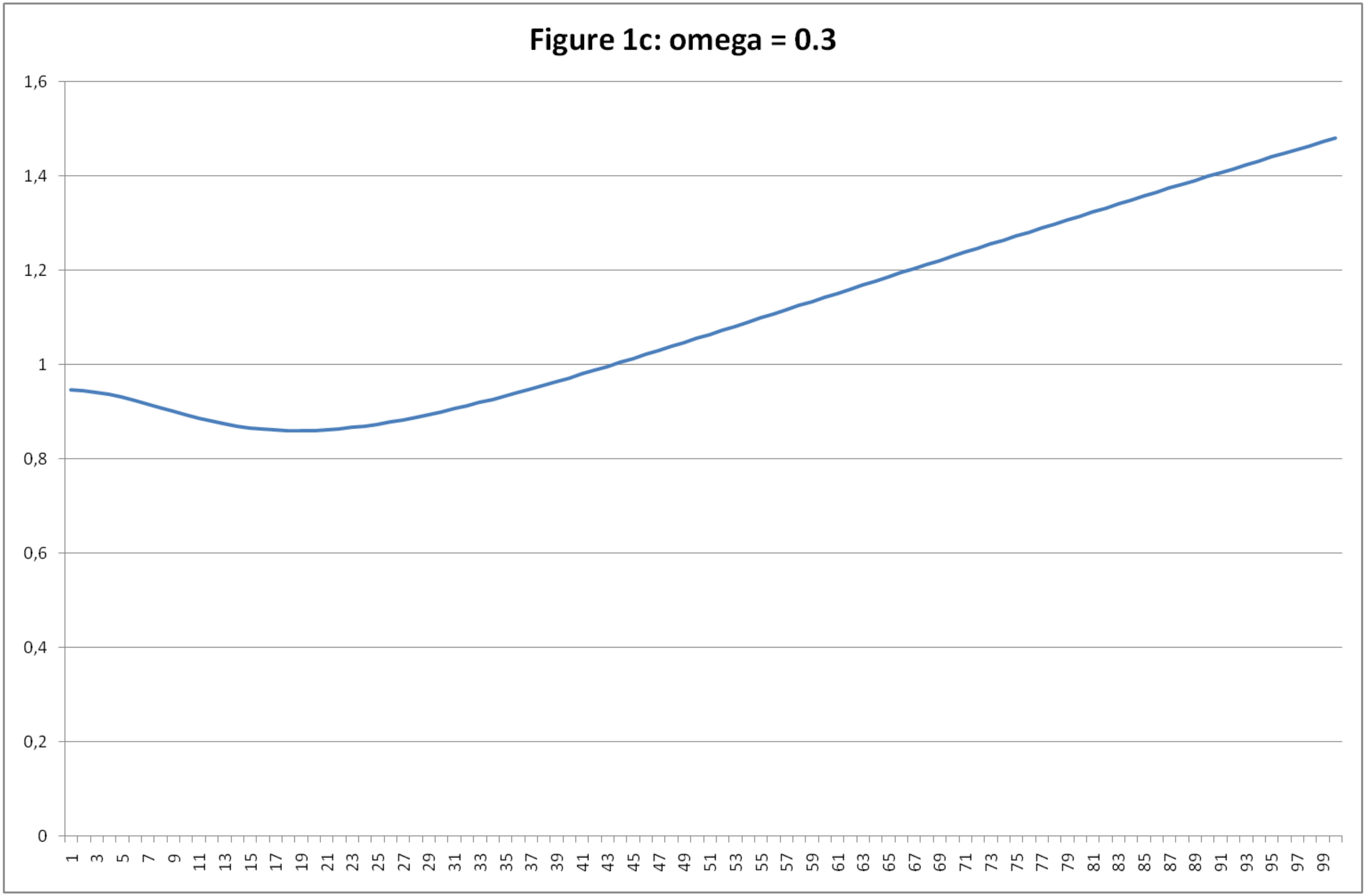
Figure 1c: omega = 0.3
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