Dynamic (In)consistent Antitrust Enforcement and Cartel Infringement Antitrust and Cartel: A Differential Game Approach

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Introduction

Dynamic Inconsistency

- **Dynamic (time) inconsistency** demonstrates a situation where a decision-maker's best decision at one point is not necessarily consistent with what is preferred at another point in time. Therefore, optimality principle does not remain optimal at any instant of time throughout the game along the equilibrium path.
- The inconsistency is primarily about **commitment** and credible threats which are important issues in law enforcement literature.
- We incorporate **dynamic (in)consistency** and **(non)commitment** in the (simultaneously à la **Nash**, or hierarchically à la **Stackelberg**) interaction between the **antitrust authority** (AA) and a **firm**, over an infinite horizon.

Introduction

Contributions

- The novelties of this paper are twofold

1. Theorists mostly assume that the AA **commits ex ante** to some probability of investigation (e.g. Motta and Polo, 2003) or the AA investigates according to an arbitrary **rule of thumb** (Harrington, 2005). We relax this assumption in feedback solution in which, an **AA has no commitment** on his auditing strategy and could simply revise its policy based upon not only acquired information at any instant of time but also the history of actions.
2. This paper goes beyond most of literature with regard to the cartel **penalty scheme** which is not only proportional to the current infringement degree but also to its record as well.

Introduction

Main Results

1. In the Nash solution, the **probability of auditing** is decreasing with the **fine structure parameters** and the **cost of investigation** whereas in the Stackelberg, change of the **probability of auditing with respect to these parameters** is ambiguous.
2. Contrary to the literature that provokes commitment for the authorities, we found that a credible **commitment** of authority on the frequency of use of this procedure **may not necessarily enhance the efficiency** of the enforcement of the competition law.

Related Literature

Dynamic Games

- The aim of **Fent et al. (1999)** is to discover the optimal intertemporal strategy of a profit maximizing offender under a given, static punishment policy.
- In **Fent et al. (2002)**, the Fent et al. (1999) framework was extended, considering two players, namely the authority and the offending individual.
- **Motchenkova (2008)** analyze a differential game of the interactions between a firm and the AA. It turns out that **full compliance behavior is not sustainable as a Nash Equilibrium under EU and US legislation** and penalty system which completely deters cartel formation is an increasing function of the degree of offence and negatively related to the probability of law enforcement.
- Our analysis is technically close to **Fent et al. (1999, 2002)** and **Motchenkova (2008)**.

Model

Control Variables

- The aim of the firm is to maximize its total expected gain by choosing q whereas the probability of auditing, denoted by p, is the AA's instrument.
- We define the variable q à la Motchenkova (2008), as $q = (P - c) \,/\, (P^m - c)$, where P is the price level, P^m is the monopoly price, and c is the marginal cost.
- q denotes the **degree of price-fixing** or the **market power** of the firm and $q \in [0, 1]$.
- $P^m = (1 + c)/2$, $\Pi := (P^m - c)^2 = (1 - c)^2/4$ is monopoly profit, with linear inverse demand $P = 1 - Q$, the producer surplus is $\pi(q) = \Pi q(2 - q)$, the net loss in total social welfare is $NLSW(q) = \Pi q^2/2$, and the consumer surplus is $CS(q) = \Pi(2 - q)^2/2$.

Model
State Variable

$$\dot{x}(t) = q(t)p(t) - \delta x(t), \quad x(t_0) = x_0. \tag{1}$$

- The state variable, $x(t)$, has two potential interpretations:

1. The **record of past crimes** (antitrust perspective)
2. The **level of experience** in forming collusion (firm perspective)

- Former crimes are only considered for a limited period and the authority would count infringements that are in the distant past, less seriously.

Model

Penalty Scheme

- The penalty scheme resembles the main feature current **European antitrust laws**: the base penalty is proportional to not only the current gravity of the infringement, $q(t)$, but also its criminal record, $x(t)$:

$$S(q, x) = k\Pi q(t) + \varphi x(t).$$

- **Additivity** makes it possible to punish a firm which has not violated the law in the current period but had participated in the cartel in some of the previous periods.

Model

Firm Problem

- The objective of the firm is to maximize the discounted summation of expected profit

$$J_F = \int_{t_0}^{\infty} e^{-rt} \left[\Pi q(2-q) - p(k\Pi q + \varphi x)\right] dt, \qquad (2)$$

subject to (1), where $r \geq 0$ denotes the discount rate.

- In practice, there are **legal restrictions** on the severity of cartel punishment, k.

1. The AA should tolerate some minor violations of competition law: for small value of q, the firm should always make some narrow profit. Hence, k should be small enough to ensure $kp < 2$.
2. The AA should not be that lenient with respect to anti-competitive behaviors: k should be large enough to make $kp > 1$.

Model

Authority Problem

- The cost of law enforcement is quadratic, i.e. $Np^2(t)$.
- The aim of the AA is to maximize welfare or equivalently to minimize the social loss. The objective functional is the discounted summation of expected profit:

$$J_A = \int_{t_0}^{\infty} e^{-rt} \left[-\frac{1}{2}\Pi q^2 - Np^2 + p(k\Pi q + \varphi x) \right] dt, \qquad (3)$$

subject to (1).

Nash Game

Firm Best Responses

Lemma

Given $p(t) := \phi$, There is a unique stationary open-loop and feedback Nash equilibria for the firm problem

$$q^o = \frac{(r+\delta)(2-k\phi)\Pi - \phi^2\varphi}{2\Pi(r+\delta)}, \tag{4}$$

$$q^f = \frac{(1+r+\delta)(2-k\phi)\Pi - \phi^2\varphi}{2\Pi(1+r+\delta)}. \tag{5}$$

- The fact that $\partial q^o/\partial r = \varphi\phi^2/2\Pi(r+\delta)^2 > 0$, yields $q^f > q^o$.

Nash Game

Firm Best Open Responses (proof)

Proof.

The current value Hamiltonian is

$$H_F(q, x, \lambda) = e^{-rt}\left\{\Pi q(2-q) - \phi\left(k\Pi q + \varphi x\right) + \lambda(q\phi - \delta x)\right\}.$$

We derive the adjoint equation as $\dot{\lambda} = r\lambda - \partial H_F/\partial x = \varphi\phi + (\delta + r)\lambda$ and the optimal control $q^o = \left(2\Pi - k\Pi\phi + \lambda\phi\right)/2\Pi$. Differentiating q^o, and given $\dot{x}$ yields

$$\begin{bmatrix} \dot{q} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \delta + r & 0 \\ \phi & -\delta \end{bmatrix} \begin{bmatrix} q \\ x \end{bmatrix} + \begin{bmatrix} \frac{\phi[\varphi\phi + k\Pi(\delta+r)]}{2\Pi} - (\delta + r) \\ 0 \end{bmatrix}.$$

Since the determinant is negative, the solution $q^o = \left[(r+\delta)(2-k\phi)\Pi - \phi^2\varphi\right]/2\Pi(r+\delta)$, is a saddle. We could show also that $q^o < q^m = 1$. $\square$

Nash Game

Firm Best Feedback Responses (proof)

Proof.

We should guess a value function, $V(x) = a_F x^2/2 + b_F x + c_F$, where a_F, b_F and c_F are unknown coefficients. The feedback solution of firm must satisfy the HJB equation,

$$rV(x) = \max\left\{\Pi q(2-q) - \phi(k\Pi q + \varphi x) + \frac{\partial V(x)}{\partial x}\dot{x}(t)\right\}. \quad (6)$$

This gives $q^f = \Pi(2 - k\phi) + b_F\phi + \phi a_F x/2\Pi$. We can substitute for q^f into (6) and collect for x, to get $\beta_1 x^2 + \beta_2 x + \beta_3 = 0$. Hence, coefficients β_1, β_2 and β_3, should be simultaneously zero which give us a_F, b_F and c_F, and finally

$$q^f = \frac{(1+r+\delta)(2-k\phi)\Pi - \phi^2\varphi}{2\Pi(1+r+\delta)}.$$

$\square$

Nash Game

Authority Best Responses

Lemma

If auditing is costly enough, $N > \grave{N} := \frac{\varphi q(2\delta+r)}{2\delta(\delta+r)}$, given the firm's choice of $q(t) := \psi$, there is a unique open-loop and feedback Nash equilibria for the antitrust problem:

$$p^o = \frac{k\Pi\delta\psi(\delta + r)}{2\delta N\,(r+\delta) - \varphi\psi\,(r+2\delta)}, \tag{7}$$

$$p^f = \frac{\delta k\psi\Pi(1+r+\delta)}{2\delta N(1+r+\delta) - \psi\varphi\,(1+r+2\delta)}. \tag{8}$$

- Since $\partial p^o/\partial r = -\frac{\varphi\psi^2\delta^2\Pi k}{[2\delta N(r+\delta)-\varphi\psi(r+2\delta)]^2} < 0$, we have $p^o > p^f$.

Nash Game

Nash Equilibrium

Proposition

There is a unique equilibrium in Nash game in which the firm would play according to the open-loop solution whereas the antitrust play the feedback solution.

- This result is consistent with the result of **Cellini and Lambertini (2004)**, though in different setting. They investigate a dynamic oligopoly game with price adjustments and show that firms prefer the open-loop equilibrium to the feedback equilibrium, and the latter to the closed-loop equilibrium. The opposite applies to consumers.

Nash Game

Nash Equilibrium Proof

Proof.

If we substitute for (7) and (8) into the $\pi_A(q, p, x)$, we would get

$$\pi_A^o(q, p^o, x) = \frac{1}{2}\Pi q^2 + \left(q\varphi - N\left(r+\delta\right)\right)\left(\delta + r\right)\left(\frac{k\Pi\delta q}{\Omega}\right)^2$$

$$\pi_A^f(q, p^f, x) = \frac{1}{2}\Pi q^2 + \left(1+r+\delta\right)\left(q\varphi - N(1+r+\delta)\right)\left(\frac{\delta k\Pi q}{\Phi}\right)^2$$

where $\Omega := 2\delta N\left(r+\delta\right) - \varphi q\left(r+2\delta\right)$ and $\Phi := 2\delta N(1+r+\delta) - \varphi q((1+r+2\delta))$. Since $\partial\pi_{AA}^O/\partial r = \varphi^2 q^4 \delta^2\Pi^2 rk^2/\Omega^3 > 0$, we have $\pi_A^o < \pi_A^f$. Similarly, by substituting for (4) and (5) into the $\pi_F(q, x)$, we could show that $\pi_F^o > \pi_F^f$. $\square$

Nash Game
Uniqueness Proof

Proof.

We substitute for (7) into (4), that yields a polynomial of q,

$$G(q) \; : \; = 2\Pi(r+\delta)q - \Pi(2 - \frac{k^2\Pi\delta q(1+\delta+r)}{\Phi})(r+\delta) + \varphi\left(\frac{k\Pi\delta q(1+\delta+r)}{\Phi}\right)^2.$$

Since q is bounded between 0 and 1 and $G(0) = -2\Pi(r+\delta) < 0$,

$$G(1) > k^2\Pi^2\delta(r+\delta+1)(r+\delta)^2(2N\delta-\varphi)/\Phi^2 > 0,$$

$$\frac{\partial G}{\partial q} > \frac{2\Pi}{\Phi^2}\left[\begin{array}{c} (r+\delta+1)^2\left(\begin{array}{c}\Omega(r+\delta+1) \\ +q\varphi\delta(2r+2\delta+3)\end{array}\right) \\ \Pi N k^2\delta^2 + (r+\delta)\Omega^3 \end{array}\right] > 0,$$

there is an unique solution for q and consequently for p. $\square$

Nash Game
Time Inconsistency

- Under some circumstances, the open-loop solution is also time consistent.
- To be able to accomplish comparison with respect to commitment, we should demonstrate that our open-loop solution is not time consistent.

Corollary

The open loop Nash equilibria of this game are not Markov perfect.

Proof.

For the open loop Nash equilibria to be Markov perfect we should have $\partial H_A/\partial^2 x = \partial H_F/\partial^2 x = 0$, and $\partial H_A/\partial x \partial p = \partial H_F/\partial x \partial q = 0$. The second condition is not satisfied since $\partial H_A/\partial x \partial p = -\varphi \neq 0$. Therefore, our open loop solutions are not time consistent. □

Nash Game

Comparative Assessment

Corollary

The **rate of law enforcement** *is* **decreasing** *with respect to the* **cost of auditing** *and the* **fine parameter**. *The* **infringement degree** *is* **decreasing** *with the* **fine structure parameters** *and* **increasing** *with respect to the* **auditing cost**.

Proof.

$$\frac{\partial q}{\partial k} = -\frac{\partial G/\partial k}{\partial G/\partial q} < 0.$$

We could show analogously for other parameters. $\square$

The Stackelberg Game

Difficulty!

- In a Stackelberg solution of differential games, the leader has only instantaneous stagewise advantage over the follower.
- Differential games in which the players use feedback strategies are hard to solve for Stackelberg equilibria.
- **Basar and Olsder (1982, pp. 315)** already noted that "*such decision problems cannot be solved by utilizing standard techniques of optimal control theory [. . .] because the reaction set of the follower cannot, in general, be determined in closed form, for all possible strategies of the leader, and hence the optimization problem faced by the leader on this reaction set becomes quite an implausible one*".
- **Dockner et al. (2000, pp 134)**, also admit that "*the analysis of such an equilibrium in a differential game may lead to considerable technical difficulties*".

The Stackelberg Game

Feedback Stackelberg equilibrium

- Since we have already showed that the firm would play according to the open-loop solution whereas the antitrust play the feedback solution, we solve for the Stackelberg equilibrium in which the **leader** is the **firm** with **open-loop** strategy and the **follower** is the **authority** with **feedback** strategy.

Proposition

There is a unique Stackelberg equilibrium in this game in which the firm (leader) would play according to the open-loop solution whereas the antitrust (follower) play the feedback solution.

The Stackelberg Game

Feedback Stackelberg equilibrium

Proof.

The leader takes into account the follower's best reply
$p^f = \left[k\Pi q(\delta + r + 1) + (2\delta + r + 1)\varphi x\right] / 2N(\delta + r + 1)$:

$$H(t) = e^{-rt} \left\{ \begin{array}{c} \Pi^m q(2 - q) - \left[k\Pi^m q + \varphi x\right] \frac{k\Pi q(\delta+r+1)+(2\delta+r+1)\varphi x}{2N(\delta+r+1)} \\ +\lambda \left[q \frac{k\Pi q(\delta+r+1)+(2\delta+r+1)\varphi x}{2N(\delta+r+1)} - \delta x\right] \end{array} \right\}$$

This give rise to $\dot{\mu}(t) = \mu r - \partial H / \partial x$. The solution, contrary to the open-loop case, is stable,

$$\begin{bmatrix} \dot{\mu} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} r - \frac{\Phi}{2N(r+\delta+1)} & \frac{\varphi^2(r+2\delta+1)}{N(r+\delta+1)} \\ 0 & \frac{(2\delta+r+1)\varphi q-\delta}{2N(\delta+r+1)} \end{bmatrix} \begin{bmatrix} \mu \\ x \end{bmatrix} + \begin{bmatrix} \frac{\Pi kq\varphi(2r+3\delta+2)}{2N(r+\delta+1)} \\ \frac{k\Pi q^2(\delta+r+1)}{2N(\delta+r+1)} \end{bmatrix}$$

$\square$

The Stackelberg Game
Uniqueness Proof

Proof.

It is enough to illustrate that

$$q = \frac{4N\Pi\left(1+r+\delta\right)+kx\Pi\delta\varphi - x\varphi\left(1+r+2\delta\right)\left(2k\Pi-\mu\right)}{2k\Pi\left(1+r+\delta\right)\left(k\Pi-\mu\right)+4N\Pi\left(1+r+\delta\right)},$$

could only has one solution. The simple re-arrangement provide us with

$$\begin{aligned} F(q) \quad : \quad &= q\left[2k\Pi\left(1+r+\delta\right)\left(k\Pi-\mu\right)+4N\Pi\left(1+r+\delta\right)\right] \\ &\quad -\left[4N\Pi\left(1+r+\delta\right)+kx\Pi\delta\varphi - x\varphi\left(1+r+2\delta\right)\left(2k\Pi-\mu\right)\right], \end{aligned}$$

which is $F(0) = -4N\Pi\left(r+\delta+1\right) < 0$, $F(1) > 0$ and $\partial F/\partial q > 0$. Therefore, there is just one q which could make $F(q) = 0$. Given this q, there would be also just one solution for p. □

The Stackelberg Game

Comparative Assessment

Proposition

Under the Stackelberg equilibrium, The infringement degree is decreasing with and fine structure parameters and increasing with respect to the auditing cost, whereas changes of the rate of law enforcement with respect to the cost of auditing and the fine parameter are ambiguous and depend on the elasticities of the infringement degree with regard to these parameters.

The Stackelberg Game

Comparative Assessment

Proof.

Since $p^f = [(k\Pi q(\delta + r + 1) + (2\delta + r + 1)\varphi x)] / 2N(\delta + r + 1)$, if $\frac{q/k}{\partial q/\partial k} = \frac{1}{\epsilon_{q,k}} < \grave{\epsilon} := -4N^2\delta^2 (r + \delta + 1)^2 / \Phi^2$, then

$$\partial p/\partial k = \Pi \left[q\Phi^2 + 4N^2 k\delta^2 (r + \delta + 1)^2 \partial q/\partial k \right] / 2N\Phi^2 < 0$$

where $\grave{\epsilon}$ in absolute value is less than one. Similarly if $\epsilon_{q,N} := \frac{\partial q/\partial N}{q/N} < 1$ then

$$\partial p/\partial N = 2k\Pi\delta^2 (r + \delta + 1)^2 (N\partial q/\partial N - q) / \Phi^2 < 0$$

and If $\frac{\partial q/\partial \varphi}{q/\varphi} > -1$ then

$$\frac{\partial p}{\partial \varphi} = \frac{k\Pi\delta (r + \delta + 1) \left[2N\delta (r + \delta + 1) \partial q/\partial \varphi + q^2 (r + 2\delta + 1) \right]}{\Phi^2} < 0$$

Conclusion

- The **infringement degree** is **declining** with **penalty structure parameters** and **increasing** with respect to the **cost of auditing** under both Nash and Stackelberg.
- The **probability of auditing** is decreasing with the **fine structure parameters** and the **cost of investigation** in the Nash solution but not necessarily in the Stackelberg one.
- **Firms** have **higher cartel intensity** under the **feedback** solution rather than the open-loop equilibrium. On the contrary, the **open-loop** solution give rises to higher **antitrust enforcement** than the feedback equilibrium. Hence, from the **firms' viewpoint**, the **open-loop solution is preferred** to feedback equilibrium, whereas the **feedback equilibrium is socially preferred** to the open-loop equilibrium.