The interplay between network investment and content quality in the Internet

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Abstract

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1 Introduction

The increasing development of the Internet network gave rise to a huge regulatory debate these last years. The most important issue is certainly on the neutrality of the Internet and its impacts on the incentives to invest of the network operators and content providers both in network infrastructures and quality of services. The debate over net neutrality raises many questions about how should be organized relationships between network operators and content providers, manely in terms of pricing and quality of access to broadband transmission services (Schuett (2010) gives a recent overwiew of these issues). One of main question is the condition under which regulators should allow network operators to adopt traffic management practices to avoid congestion and ensure a sufficient quality of service to content providers for offering their services. Recently, on september 2011, the FCC released its final net neutrality rules for preserving an open Internet and, stressed the need for transparency in network management practices and reasonable discrimination in

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transmitting network traffic. In Europe, the Commission should give its recommendations to ensure the open Internet in late 2012.

In Europe, close to the net neutrality the regulatory debate is also focused on the investment in Next Generation Networks (NGNs). The question is how give incentive to the network operators to invest in new commication infrastructures and then to migrate from the copper network (the old technology) to the fiber network (the new technology). The economic literature focused on this topic had mainly analysed the impacts of the access pricing rules on the incentives of operators to roll-out new infrastructures. How to manage the coexistence of the old and the new technologies is certainly the main issue for National Regulatory Authorities (NRAs). The interplay between investment and access price has already been studied in the economics literature, for instance, by Brito, Pereira and Vareda (2010), and more recently by Bourreau, Cambini and Dogan (2011) and Bourreau, Cambini and Hoernig (2012) in models that introduced directly the issue of the technological migration and the balancing effects of the network access price on the incentives to invest in the new technology.

The contribution closest to ours are those that have modelled the key impact of network neutrality on the investment of the network operators. The first rigourous theoretical analysis of net neutrality can be found in Economides and Tag (2007)[1]. In a two-sided framework, this paper analyses a model where network operators can charge CPs for traffic termination to consumers. They show that net neutrality, viewed as a no access fees regime, can greatly improve the consumer surplus but they do not consider investment of network operators and innovation of CPs. Economides and Hermalin (2010) consider there is a limited bandwidth to be allocated between CPs and look at the ISP's incentive to invest in more bandwidth. Cheng et al. (2011), Choi and Kim (2010), and Kramer and Wiewiorra (2010) study a model of queuing theory to modelize congestion on the Internet. They show that priority pricing can be welfare enhancing in short-run and can increase the ISP's incentive to invest in network infrastructure. While all these contributions give interesting insights of the impacts of net neutrality on network operators and CPs behaviors, none of these papers directly analyzed the interplay between the investment decision of the network operators and the investment in content quality of CPs. Yet, there exists a strong relationship between both that should be taking into account when

[1]Hermalin and Katz (2007) analysed net neutrality as a restriction on the product line from the network operator but not consider the Internet traffic.

analyzing the potential impacts of net neutrality on infrastructure investment of network operators.

The aim of this paper is to contribute to the question of investment in the NGNs by focusing on the interplay between infrastructure investment from the network operators and investment in content quality from the content providers. Usually, Internet broadband is viewed as two-sided market consisting of consumers on one side and content providers on the other. The interplay between both sides pass through the way prices are set on both sides. However, the consumers willigness to pay to access the network depends on the number of content providers but also on the quality of contents they offer. On the other hand, the incentives to invest of network operators crucially depends on how they can price the access to the new technology on both sides of the platform. As the consumer willingness to pay is usually an increasing function of the quality of contents, the network operator's incentive to invest should be potentially even stronger when the quality of content is high. That is certainly a part of the mechanisms that can increase the incentive to invest in a new technology for the network operator. The reverse is certainly true, and the incentive to upgrade the quality of content for the content providers should be also increasing with the quality of the network infrastructure.

The remainder of the paper is structured as follow. Section 2 introduces the basic model. Section 3 presents the equilibrium outcomes and give some interesting comparative statics. Section 4 analyses the investment decision of the network operator and how its investment can be impacted by the potential expansion for content quality that can come from the new technology. Section 5 concludes. All proof are relegated to the appendix.

2 The model

We introduce a model in which a monopolistic network operator (hereafter, the ISP) sells access to consumers and to Content Providers (CPs). The ISP offers two network technologies, an old technology (the copper) and a new technology (the so-called next generation access fibre network, NGN). Content providers are horizontally differentiated and compete à la Hotelling. This section presents the key assumptions of the model.

2.1 The monopolistic platform

We consider a model in which a monopolistic network operator (ISP) sells Internet access to consumers and provides access to content providers. Consumers can get the contents of CPs from Internet access.

We assume that the ISP possibly offers two network technologies, an old technology (copper) and a new one (fiber). We note these two technologies $j = c$ and $j = f$, respectively for the copper (old technolgy) and the fiber (the new technology). Getting access from copper or fiber allows end-users to get different content qualities. So, when end-users subscribe to the new technology (fiber), they can get access to contents of a higher quality. We then consider two markets, one in which the ISP offers both technologies and the other in which it offers the old technology only. Consumers are split between the two markets as follows: a fraction λ of consumers is in the market in which both the copper and the fiber technologies are offred (market U), and a fraction $1 - \lambda$ of consumers is in the other market, where only the copper technology is offred (market R).[2]

The ISP sells access to the old technology to consumers at price p_c and to the new one at price p_f. We then assume non discriminatory price between both markets R and U as consumers can get access to the old technology at a same price, p_c. The ISP possibly collects from each content provider i a unit price s_{ji} *per* end-user connected to the platform through technology $j = c, f$. We assume non-discriminatory price between both CPs, hence $s_{ci} = s_c$ and $s_{fi} = s_f$. Finally, we assume that the marginal cost of providing the access to both sides of the platform is the same whatever the access technology and the market, and we normalize it to 0.

2.2 Content providers

We consider a competition between two content providers differentiated *à la Hotelling*. For each content ($i = A, B$), consumers perceive a low or a high quality depending on whether they get access to the old technology or to the new one. We note θ_i the perceived quality of content that CPi offers when consumers subscribe to the old technology. Subscribing to the new technology allows consumers to get a higher quality for content i, that is noted $\overline{\theta}_i$. That is, the new technology provides more functionalities for applications and then

[2]This market segmentation can represent regional markets, for example a rual area and an urban area where the asymmetry lies into the deployment of the new access technology.

consumers benefit from a higher quality. We define the degree of quality differentiation, for contents θ_i and $\overline{\theta}_i$, as the difference between the higher and the lower quality for each type of contents. That is to say, the degree of quality differentiation is given by $(\theta_A - \theta_B)$ for contents consumed with the old technology and, by $(\overline{\theta}_A - \overline{\theta}_B)$ for ones consumed with the new technology. Moreover, we assume that the quality for content A is higher than the quality for content B whatever the access technology ($\theta_A > \theta_B$ and $\overline{\theta}_A > \overline{\theta}_B$) and that new technology always allows end-users to get a higher content quality, $\overline{\theta}_B > \theta_A$.

We assume that both content providers use the same technologies and we normalize the marginal cost to 0. CPi sells content at a unit price p_{ci} when consumers get it through the old technology and at price p_{fi} when consumers use the new technology. Contents consumed by consumers through the old technology can be considered as basic contents while contents consumed through the new technology can be viewed as premium contents. So, we then assume in the following that end-users bear no other costs than the broadband access subscription fee when they get the old technology and thus they can consume basic contents for free (i.e. $p_{ci} = 0$). Finally, content providers also receive revenues from online advertisements and we note a the per-click advertisement revenue.

2.3 Consumers

We assume a continuum of end-users with heterogeneous preferences over network technologies $y \in [0,1]$ and over the contents $x \in [0,1]$. In market U, access technologies are located at the two extremities of the segment, namely at $y = 0$ for the old technology and $y = 1$ for the new one. In market R, the ISP only offers the old techonology which is located at $y = 0$. x is the location of the consumer in a fixed location Hotelling model between contents A and B where A is exogeneously located at $x = 0$ and B at $x = 1$. We assume that consumers cannot buy both contents.

Given γ the transportation unit cost in the choice of network technology, the consumers' utility is set to $v_c = u_c - p_c - \gamma y$ if they buy the old technology, $v_f = u_f - p_f - \gamma(1-y)$ if they buy the new technology, and 0 in the case they do not buy network access. We assume that the gross utility for joining the old technology is lower than for the new technology, $u_c < u_f$. Given t the transportation unit cost in the choice of content, consumers who buy technology j ($j = c, f$) get an excess utility $v_{jA} - v_j = q_{jA} - p_{jA} - tx$ if they buy content A, and utility $v_{jB} - v_j = q_{jB} - p_{jB} - t(1-x)$ if they buy content B, where

$q_{ci} = \theta_i$ and $q_{fi} = \overline{\theta}_i$ with $i = A, B$. Hereafter, we maintain the following assumptions.

Assumption 1. 1.1) $\theta_A - \theta_B < t$; 1.2) $\overline{\theta}_A - \overline{\theta}_B < 3t$; 1.3) $\theta_A + \theta_B > t$; 1.4) $\overline{\theta}_A + \overline{\theta}_B > 3t$

Assumption 1 implies that both contents are consumed (assumptions 1.1) and 1.2)) and content market is fully covered (assumptions 1.3) and 1.4)) whatever the contents offered by CPs (basic contents or premium contents) and whether CPs get positive revenues from online advertisements or not ($a \geq 0$). The two first conditions state that both firms can sell a strictly positive quantities of their contents (basic and premium contents). That is, CP$_A$ does not offer a so much better quality than CP$_B$ (the degree of quality differentiation $\theta_A - \theta_B$ and $\overline{\theta}_A - \overline{\theta}_B$ are not too high). The two last conditions state that the total gross utility ($\theta_A + \theta_B$ and $\overline{\theta}_A + \overline{\theta}_B$) should be high enough to ensure that the marginal consumer between content A and B has a strictly positive excess utility (for both type of contents).

Let us assume that consumers learn their preferences over contents only after having bought the network access. That is to say, before having access to the network, end-users have not yet information about content characteristics and they do not know their preference over contents. This effect can be viewed as a kind of learning-by-doing effect. Hence, the utility that consumers get from network technologies depends *ex-ante* on the expected utility from buying content A or B respectively. We note, EU_j the expected utility derived from contents when consumers get the technology j.

2.4 Market shares, surplus and welfare

We consider first the market shares for contents and network technlogies, the profits for the ISP and the CPs and finally the consumers surplus and the welfare.

Market shares. In the following, we determine marginal consumers both for CPs and the ISP. Let us first start by marginal consumer between contents A and B and consider markets R and U.

In each market R and U, consumers choose to get content from CP$_A$ or CP$_B$ and their excess utility for contents depend on the network technology. Consider consumers who have bought the network technology j, the marginal consumer between content A and B is such that $v_{jA} - v_j = v_{jB} - v_j$, and is explicitly given by:

$$q_{jA} - p_{jA} - tx_j = q_{jB} - p_{jB} - t(1 - x_j)$$

We note $\widetilde{x}_c$ and $\widetilde{x}_f$ market shares for content provider A respectively for basic contents and premium contents. We consider full market coverage for contents. Hence, market shares for content provider B are respectively $1-\widetilde{x}_c$ and $1-\widetilde{x}_f$. Market shares for CPs are then determined as follows:

$$\widetilde{x}_j = \frac{1}{2} + \frac{p_{jB} - p_{jA}}{2t} + \frac{q_{jA} - q_{jB}}{2t} \tag{1}$$

Finally, the utility that consumers are expected to get from the purchase of contents when they subscribe to the network technology j is:

$$EU_j = \int_0^{\widetilde{x}_j} (q_{jA} - tx)dx + \int_{\widetilde{x}_j}^1 (q_{jB} - t(1-x))dx \text{ with } j = c, f \tag{2}$$

Let us now determine market shares for network technologies. When consumers choose the network technology that they want access, they base their decisions on the expected utility from the content consumption. We note $w_j = u_j + EU_j$, the total gross (expected) utility that consumers get from contents when they join technology j. Consumers' expected utility is $v_c + EU_c = w_c - p_c - \gamma y$ if they get access to the old technology, and $v_f + EU_f = w_f - p_f - \gamma(1-y)$ if they get access to the new technology.

Explicitly, the marginal consumer's decision, for consumers in market R, is based on $w_c - p_c - \gamma y = 0$. The location of the indifferent consumer $\widehat{y}$ is then:

$$\widehat{y} = \frac{1}{\gamma}(w_c - p_c) \tag{3}$$

In market U, the ISP owns both network technologies. The monopolist ISP sets prices p_c and p_f such that consumer $\widetilde{y}$ who is indifferent between buying a network technology (copper or fiber) and not buying is given by $v_c + EU_c = v_f + EU_f$. Explicitly, the marginal consumer's decision for consumers in market U is based on:

$$\widetilde{y} = \frac{1}{2} + \frac{p_f - p_c}{2\gamma} + \frac{w_c - w_f}{2\gamma} \tag{4}$$

Hence, market shares for the ISP in market R and U are deduced from (3) and (4).

Profit functions. Content providers get profits from each market (R and U). We first determine their unit profit function for a consumer who subscribes either to the old technology or the to the new one. From the CPs' market shares determined above, we can easily write the unit profits that they get from a consumer subscribing to the network technology j are:

$$\pi_{jA} = (a + p_{jA})\widetilde{x}_j - s_j \quad \text{and} \quad \pi_{jB} = (a + p_{jB})(1 - \widetilde{x}_j) - s_j \tag{5}$$

Considering now both markets, R and U. The overall profit Π_i that CP$_i$ extracts from selling its content in both markets is:

$$\Pi_i = [(1-\lambda)\widehat{y} + \lambda\widetilde{y}]\,\pi_{ci} + \lambda(1-\widetilde{y})\pi_{fi} \text{ with } i = A, B \tag{6}$$

Finally, from the ISP's market shares in U and R, we can deduced the overall ISP's market share: $\lambda\widetilde{y} + (1-\lambda)\widehat{y}$ for the old technology, and $\lambda(1-\widetilde{y})$ for the new technology. Hence, rearranging terms, the overall ISP's profit becomes:

$$\Pi_{isp} = \lambda\left[(p_c + s_c)\widetilde{y} + (p_f + s_f)(1-\widetilde{y})\right] + (1-\lambda)(p_c + s_c)\widehat{y} \tag{7}$$

Consumers surplus and welfare. Consumers get surplus from content consumption and from network technologies. For a consumer that subscribes to technology j, the expected utility from the purchase of contents is given by (2). From (4), the total consumers surplus SC_R and SC_U, respectively for markets R and U are:

$$SC_R = \int_0^{\widehat{y}} (w_c - p_c - \gamma y)\, dy$$

and, (8)

$$SC_U = \int_0^{\widetilde{y}} (w_c - p_c - \gamma y)\, dy + \int_{\widetilde{y}}^1 (w_f - p_f - \gamma(1-y))\, dy$$

The overall consumer surplus is $SC = (1-\lambda)SC_R + \lambda SC_U$ and the welfare is the sum of the overall consumer surplus and the overall CPs and ISP's profits, $W = SC + \Pi_A + \Pi_B + \Pi_{isp}$.

In the following, we consider a two stages game: in the first stage, the ISP sets prices for both network technologies (p_c and p_f) and consumers buy access; in the second stage, consumers learn their preference over contents, content providers set their unit price for contents (p_{fA} and p_{fB}) and consumers buy contents.

3 The Equilibrium

In this section, the game is solved by backward induction. Firstly, we consider CPs' profit for a consumer whatever the market (U and R) and afterwards we will look at the network technology choices, and, we derive the ISP and CPs' profits considering both markets U and R. For the sake of convenience, we note $p_{fA} = p_A$ and $p_{fB} = p_B$.

3.1 Equilibrium in Stage 2

Assuming subgame perfect equilibrium, we start by solving Stage 2. In this stage, consumers buy contents taking as given their network technology choices. We consider full market coverage for contents. Hence, CPs' market shares for consumers who have bought network technology j are given by (1).

First, consider consumers who have joined the old technology. Joining the old technology allows consumers to get basic contents for free ($p_{ci} = 0$), and in this case, the perceived quality from content i is $q_{ci} = \theta_i$.

The price for content i is $p_{ci} = 0$ when consumers join the old technology. Then, there is no optimization problem to be solved for CPs. The total gross utility $w_c^* = u_c + EU_c^*$ that they expected to get from the purchase of both contents is then derived from:

$$EU_c^* = \int_0^{\widetilde{x}_c} (\theta_A - tx)dx + \int_{\widetilde{x}_c}^1 (\theta_B - t(1-x))dx \quad \text{where} \quad \widetilde{x}_c^* = 1/2 + (\theta_A - \theta_B)/2t$$

Then,

$$EU_c^* = \frac{(\theta_B + \theta_A)}{2} + \frac{(\theta_A - \theta_B)^2}{4t} - \frac{t}{4} \tag{9}$$

The unit profit π_{cA}^* and π_{cB}^* (*per* consumer) for content providers are:

$$\begin{aligned} \pi_{cA}^* &= a\widetilde{x}_c^* - s_c = a\left(\frac{1}{2} + \frac{\theta_A - \theta_B}{2t}\right) - s_c \\ \pi_{cB}^* &= a(1 - \widetilde{x}_c^*) - s_c = a\left(\frac{1}{2} - \frac{\theta_A - \theta_B}{2t}\right) - s_c \end{aligned} \tag{10}$$

Let's now consider premium contents. When consumers subscribe to the new technology, they pay p_f and the contents perceived quality is $q_{fi} = \overline{\theta}_i$. Under full market coverage for contents, we obtain the optimal price for each CP by maximizing their profit:

$$\max_{p_A} \pi_{fA} = (a + p_A)\widetilde{x}_f - s_f \quad \text{and} \quad \max_{p_B} \pi_{fB} = (a + p_B)(1 - \widetilde{x}_f) - s_f \tag{11}$$

The following Lemma gives the resulting price equilibrium from the CPs' maximization program.

Lemma 1 *The equilibrium prices for premium contents are* $p_A^* = t - a + \left(\overline{\theta}_A - \overline{\theta}_B\right)/3$ *and* $p_B^* = t - a - \left(\overline{\theta}_A - \overline{\theta}_B\right)/3$.

The proof is straithforward from (11). Lemma 1 states a standard result for a Hotelling equilibrium when market is fully covered with both contents. Equilibrium prices differ from the marginal cost and are equal to $t-a$ when contents qualities are the same ($\overline{\theta}_A = \overline{\theta}_B$). This reveals that t should be higher than a to ensure positive prices. Moreover, equilibrium prices increase with the degree of quality differentiation.

Substituting equilibrium prices (p_A^* and p_B^*) into (5), we obtain equilibrium profit for CP$_i$:

$$\pi_{fi}^* = \frac{\left(3t + \overline{\theta}_i - \overline{\theta}_{-i}\right)^2}{18t} - s_f \quad \text{where } i, -i = A, B \text{ and } i \neq -i \tag{12}$$

Combining p_A^* and p_B^* and (1), we obtain the equilibrium market shares: $\widetilde{x}_f^* = \left(3t + \overline{\theta}_A - \overline{\theta}_B\right)/6t$ for content A, and $1 - \widetilde{x}_f^*$ for content B.

Finally, the total gross utility $w_f^* = u_f + EU_f^*$ that consumers expected to get from the purchase of contents when they subscribe to the new technology is derived from:

$$EU_f^* = \int_0^{\widetilde{x}_f^*} (\overline{\theta}_A - p_A^* - tx)dx + \int_{\widetilde{x}_f^*}^1 (\overline{\theta}_B - p_B^* - t(1-x))dx$$

Then,

$$EU_f^* = \frac{(\overline{\theta}_A + \overline{\theta}_B)}{2} + \frac{(\overline{\theta}_A - \overline{\theta}_B)^2}{36t} - \frac{5t}{4} + a \tag{13}$$

Lemma 2 *Suppose that* $\overline{\theta}_i = \alpha\theta_i$. *Premium contents provide higher expected utility than basic contents* ($EU_f^* \geq EU_c^*$) *if* α *is high enough* ($\alpha \geq \alpha_2$).

The intuition of Lemma 2 is the following. Suppose that the quality for basic contents is the same than for premium contents ($\alpha = 1$). The total gross utility from joining the old technology is then higher as consumers have to pay strictly positive prices for premium contents (if they join the new technology). Increasing α induces a raising of the degree of quality differentiation for premium contents ($\alpha(\theta_A - \theta_B)$). This leads to two opposite effects on the total gross utility from joining the new technology . First, this decreases the total gross utility since more quality differentiation (higher $\alpha(\theta_A - \theta_B)$) increases equilibrium price for the premium content (high quality content). On the other hand, more quality differentiation decreases the market share for the lower quality content (content B) and then more consumers are served with the high quality at equilibrium. This positive effect outweights the first negative effect and is even higher when the degree of quality differentiation for premium contents is significantly high (α significantly high). Hence, this leads to increase the total gross utility from joining the new technology. Consequently, when α is significantly high ($\alpha \geq \alpha_2$), the total gross utility from joining the new technology becomes higher than the total gross utility from joining the old technology.

Having calculated the outcomes of stage 2, we now proceed to stage 1, where consumers choose which technology (old/new) they get access.

3.2 Equilibrium in Stage 1

At this stage, consumers decide which technology they buy. They base their decisions on the expected utility from the content consumption given by (9) and (13). Hence, using (3) and (4) the ISP's market shares for both markets (U and R) can be deduce from:

$$\widetilde{y} = \frac{1}{2} + \frac{1}{2\gamma}(p_f - p_c) + \frac{1}{2\gamma}\left(w_c^* - w_f^*\right) \tag{14}$$

$$\widehat{y} = \frac{1}{\gamma}(w_c^* - p_c) \tag{15}$$

Several cases may arise depending on whether both markets R and U are fully covered and both technologies are offred at equilibrium in market U. We note $\Delta = w_f^* - w_c^*$ and $\nabla = w_f^* + w_c^*$. For the sake of simplicity,we assume in the following that both technologies are offered in market U[3]. That is,

$$p_f - p_c - \gamma < \Delta < p_f - p_c + \gamma \tag{16}$$

$$\nabla > p_f + p_c + \gamma \tag{17}$$

[3] We note $\widetilde{y}_c$ and $\widetilde{y}_f$ respectively, the marginal consumer between buying the old/new technolohy or not buying. Full market coverage with both technology is ensured when $\widetilde{y}_c > \widetilde{y}_f$ and $0 < \widetilde{y} < 1$.

Hence, two cases may occur according to whether broadband access market is fully covered in market R or not. Let us now consider the two cases and show some interesting results. Subscript $*$ denotes equilibrium outcomes when market R is fully covered and NC when market R is partially covered.

The ISP chooses prices p_c and p_f that maximize its overall profit (7). Its optimization problem is then $\max_{p_c,p_f}\Pi_{isp}(p_c,p_f)$. The first other conditions are:

$$\frac{\partial \Pi_{isp}}{\partial p_c} = \lambda\left[2\gamma\widetilde{y} - p_c - s_c + p_f + s_f\right] + 2(1-\lambda)\left[\gamma\widehat{y} - p_c - s_c\right] = 0$$

$$\frac{\partial \Pi_{isp}}{\partial p_f} = 2\gamma(1-\widetilde{y}) + p_c + s_c - p_f - s_f = 0$$

Equilibrium prices are then:

$$p_c^* = \frac{\lambda\gamma + (1-\lambda)(w_c^* - s_c)}{2(1-\lambda)} \text{ and } p_f^* = \frac{\gamma + (1-\lambda)\left(w_f^* - s_f\right)}{2(1-\lambda)} \tag{18}$$

The following Lemma 3 gives conditions whether market R is fully covered or not.

Lemma 3 *The ISP has an incentive to fully cover market R with the old technology when $w_c^* > \dfrac{2-\lambda}{1-\lambda}\gamma - s_c$.*

Lemma 3 states that the ISP can attract all consumers in market R when the total gross utility w_c^* derived from network access (the sum between gross utility from network access and gross utility from contents) is high enough. When the total gross utility is lower, the ISP has an incentive to partially cover the market R.

When market R is partially covered, prices are then given by (18), the ISP's market share in market R for the old technology is given by (15) and market shares for both technologies in market U are deduced from (14). The overall ISP's profit is then (7).

With full market coverage, the ISP sets the marginal consumer between purchasing the old technology or not purchasing at $\widehat{y}^{FC} = 1$. The price p_c should then not be too high to ensure the outermost consumer a non-negative utility, i.e. $w_c^* - p_c - \gamma \geq 0$. We assume that the ISP gets all the utility of the consumer located at $y = 1$. That is, the equilibrium price for the old technology is set to $p_c^{FC} = w_c^* - \gamma$.

From (14), we derive the marginal consumer in market U according to market R is fully covered:

$$\widetilde{y} = 1 + \frac{1}{2\gamma}\left(p_f - w_f^*\right) \tag{19}$$

Using (19), and substituting $\widehat{y}^{FC} = 1$ and p_c^* into (7), we deduce the equilibrium price for the new technology that is $p_f^{FC} = (\nabla + s_c - s_f - \gamma)/2$. Hereafter, we maintain the following assumptions.

Assumption 2. 2.1) $s_c - s_f - \gamma < \Delta < s_c - s_f + 3\gamma$; 2.2) $\nabla > \dfrac{(3-\lambda)\gamma}{1-\lambda} - s_c - s_f$

Assumption 2 implies that both technologies are offered in market U whatever market R is fully covered or not. These conditions are obtained considering (16) and (17) are checked at equilibrium prices (p_c^{FC}, p_f^{FC}) and (p_c^*, p_f^*).

The following Proposition 1 sums up the equilibrium prices.

Proposition 1 *Market U is fully covered with both technologies, and the equilibrium access prices for consumers are: (i) p_c^{FC} and p_f^{FC} when $w_c^* > \dfrac{2-\lambda}{1-\lambda}\gamma - s_c$ and; (ii) p_c^* and p_f^* when $w_c^* \leq \dfrac{2-\lambda}{1-\lambda}\gamma - s_c$.*

The equilibrium prices p_c and p_f result from a trade-off operated by the ISP between both markets R and U. In market U, where both technologies are offred, ISP should set carefully its prices p_c and p_f. As the ISP acts as a monopolist, he should avoid setting access price to consumers in market U that induces too much competition between both network technologies he offers. In the meantime, the ISP should set a relatively low price for the old technology if he wants to attract consumers located far from its location in market R and then serve the whole market. However, setting a low price p_c may induce too much competition between both technologies in market U. The ISP solves these contrasted effects when choosing access prices to consumers by setting them to p_c^{FC} and p_f^{FC}. Lemma 3 states that the ISP can attract all consumers in market R when the total gross utility derived from network access (the sum between gross utility from network access and gross utility from contents) is high enough. In such case, the consumer gets enough if he joins the old technology and then the ISP can attract him with a not so low price p_c^{FC}. Hence, in market U competition is not disturbed and the ISP can post a relatively high access price p_f^{FC} for the new technology. In (ii), the total gross utility (w_c^*) derived from network access in market R is not enough to allow the ISP to set a not too low unit price p_c that may attract all consumers. The ISP then posts the higher price p_c that can accomode competition in market U between both technologies and attract the most of consumers in market R. The resulting trade-off is then solved with equilibrium prices, p_c^* and p_f^*, stated in result (ii). These prices do not allow the ISP to fully covered market R but that leads to the maximization of the its overall profit. Finally, we can remark that $\frac{2-\lambda}{1-\lambda}\gamma - s_c$ is an increasing function with respect to λ. That is to say, equilibrium prices derived from result (i) stated in Proposition 1 are less likely to occur when the proportion of consumers in market U is higher. In such case, the minimum total gross utility $(u_c + EU_c^*)$ that is required is indeed even higher.

Substituting (15), (14), (10), (12) and p_c^*, p_f^*, p_c^{FC}, p_f^{FC} in (7) and (6), we obtain the equilibrium profits for the ISP and CPs whether market R is fully covered or not, noted (Π_{isp}^*, Π_i^*),$(\Pi_{isp}^{FC}, \Pi_i^{FC})$.

3.3 Comparative statics

In this section, we performe some comparative statics to analyze both the impact of discriminatory network access fee for the CPs and network investment from the ISP on equilibrium outcomes. We limit our analysis to the case where market R is non fully covered.

Lemma 4 states the impact of a discriminatory network access fee on the ISP and CPS' profit.

Lemma 4 *Suppose that $s_c = s_f$. A marginal increase of s_f around s_c: (i) increases the overall ISP's profit and; (ii) increases the consumers surplus; and, (iii) increases the overall CPs' profit if their profit per consumer for premium contents (π_{fi}^*) is high enough compared to the profit they get from basic contents (π_{ci}^*).*

Lemma 4 states that discriminatory network access price is always profitable for the ISP and can improve the CPs profit. That is the case when content providers get significantly higher profit from premium contents than from basic contents. The intuition for result (i) is simple. Starting with $s_c = s_f$, a marginal increase of the new network access price induces two effects. Firstly, it decreases the price that consumers pay to get access to the new technology. Hence, this helps the ISP to attract more consumers with the new technology. This first effect comes from consumer migration from the old to the new technology in market U and it is beneficial for the ISP. Moreover, a marginal increase of s_f improves the total access revenue that the ISP gets from CPs for the new technology as more consumers join the new technology. These two combined positive effects undoubtedly increase the overall ISP's profit. As the price to get access to the new technology decreases, the ISP market share for the new technology increases and then more consumers are served with the high content quality (premium content). This migration effect from the old to the new technology in marcket U directly increases the overall consumers surplus (result (ii)). Result (iii) states that contents providers should get a significantly high profit from premium contents to benefit from discriminatory access price (s_f higher than s_c). A slight increase of s_f by decreasing the ISP's market share for the old technology in market U decreases the total profit that CPs get from basic contents. On the other hand, the consumer migration effect, that induces by discriminatory access price, leads to increase the total profit that CPs get from premium contents. The latter effect overcompensates the former when the CPs profit per consumer for premium contents is significantly higher than the one they get from basic contents, and, that is the case whether market R is fully covered or not. In Appendix, we show that the difference between both unit profits, $\pi^*_{fi} - \pi^*_{ci}$, should be higher than the threshold defined by $w^*_f - w^*_c + \gamma$. This results then depends on how consuming premium contents increases the total gross utility for consumers compared to basic contents ($EU^*_f - EU^*_c$). That is finally depends on whether the quality of premium contents is far higher than the basic quality or not.

Let's now study the impact of a marginal investment from the ISP on outputs equilibrium. The Lemma 5 shows how equilibrium prices and market shares are impacted by a marginal network investment.

Lemma 5 *A marginal network investment from the ISP: (i) increases access prices to both technologies for consumers, p^*_c and p^*_f; (ii) remains inchanged the market segmentation between both technologies in market U, and decreases the ISP's market share for the old technology in market R.*

The intuition for results (i) and (ii) is the following. As the ISP is a monopolist who sells both technologies in market U, he should manage its prices, p_c and p_f, to mitigate the potential competition between both technologies. That is to say, faced with consumers choices, the ISP could set prices at the maximum level that limits competition between both technologies. However, the ISP is also faced with consumers choices in market R. This constrains the ISP to set a price for the old technology that is not too high to attract enough consumers in market R. This constraint in market R toward a lower price for the old technology is more stringent when the proportion $1-\lambda$ of consumers in market R is high (or λ is low). Hence, when the ISP increases the proportion λ of consumers in market U through its investment, the constraint toward a low p_c becomes less stringent and the ISP can increase both prices p_c and p_f in a same proportion to remain inchanged its market shares $\widetilde{y}^*$. However, the increasing price for the old technology reduces the ISP's market share in market R.

Propositon 2 shows how the overall profit for both CPs and the overall consumer surplus are affected by a marginal network investment.

Proposition 2 *Suppose that discriminatory network access price around $s_c = s_f$ is applied. A marginal network investment from the ISP: (i) increases the consumer surplus; and, (ii) increases the overall profit of CPs if α is sufficiently high, $\alpha \geq \widetilde{\alpha}$.*

Remember that discriminatory network access price always increases the overall consumers surplus (Lemma 4). That is, discriminatory access price allows consumers to get access to the new technology with a lower price. Hence, more consumers purchase the new technology and get the high quality content. The network investment from the ISP reinforce the positive impact on the overall consumer surplus as it raises the proportion of consumers located in market U. Increasing the proportion of consumers that can be served with the new technology, the network investment allows even more consumers to get the high content quality, and consumer surplus is even higher (result (i)). The intuition for results (ii) is also quite simple. A network investment increases the proportion of consumers that are located in the market U. Hence, market R becomes tight and a first direct effect of the marginal investment is to reduce the potential market of basic contents for both content providers and so their profits. A second effect is that investment increases access prices for consumers for both technologies (the old and the new one) and then reduce even more the proportion of served consumers in market R. On the other hand, the network investment raises the potential profit that content providers can get from market U. The total impact on the overall profit for both contents providers depends thus on whether the *per consumer* profit they get from premium contents is significantly high compared to the one they get from selling basic contents. This effect is even stronger if content providers have to pay a higher fee when they get access to the new technology than to the old technology. In Appendix, we show that when α takes sufficiently high values, the *per consumer* profit for both content providers is high enough and ensure that a marginal investment from the ISP increases their overall profits (result (ii)).

4 Network investment of the ISP

This section analyses the ISP's network investment decision. Choosing its network investment, λ, allows the ISP to increase its market coverage with the new technology. We consider that the ISP's network investment decision is a decision stage that precedes the previous analysis. Then, the ISP chooses the network investment, λ, first and sets the access price for consumers, and CPs set the unit price for their content.

For the slake of clarity, we consider hereafter that the ISP has not an incentive to sell to all consumers in market R. Hence, from Proposition 1, we consider in the following that the total gross utility derived from network access in market R is relatively low, that is $w_c^* < \dfrac{2-\lambda}{1-\lambda}\gamma - s_c$. Equilibrirum prices and profits are then given by p_c^* and p_f^*, and we define the short-run equilibrium profit for the ISP as a function of λ, noted $\Pi_{isp}^*(\lambda)$. We assume the ISP decides the network investment that maximizes its overall profit considering the cost investment function is given by $c(\lambda)$. We assume that $c(0) = 0$, $c'(.) \geq 0$, $c''(.) \geq 0$, $\lim_{\lambda \longrightarrow 0} c'(.) = 0$. Hence, the long-run overall ISP's profit is $\Pi_{isp}^*(\lambda) - c(\lambda)$.

The ISP will set the optimal investment, λ, at the point where its long-run profit is maximized. Consequently, the network investment will be such its marginal short-run profit, $\partial \Pi_{isp}^*(\lambda)/\partial \lambda$, equals its marginal cost, $c'(\lambda)$. In Appendix, we show that the marginal short-run profit for the ISP is always positive if the total gross utility from consuming the new technology is sufficiently high. In this case, the concavity of the long-run profit function is ensured by the convexity of the cost function, $c(\lambda)$. We then assume that $c(.)$ is sufficiently convex.

The following Proposition 3 gives, λ^*, the equilibrium network investment of the ISP.

Proposition 3 *Assume that $w_f^* \geq \overline{w}_f$. The inner equilibrium network investment, λ^*, is implicitly given by $f(w_c^*, w_f^*, \lambda^*) = 8\gamma(1-\lambda^*)c'(\lambda^*)$.*

The intuition of the result states in Proposition 3 is the following. Lemma 5 shows that a marginal investment for the ISP increases access prices for both technologies, p_c^*

and p_f^*, without shifting the demand segmentation in market U. On the other hand, a marginal network investment shifts downward the demand for the old technology in market R as p_c^* increases. Hence, taken as given, λ, the split of consumers between both markets R and U, a marginal network investment definitely increases the *per consumer* profit that the ISP draws from market U whereas it produces a contrasted impact on its profit from market R. We can easily show that the latter effect is negative as the positive impact of network investment on p_c^* is not high enough to balance the negative impact produced on the ISP's market share in market R. So, when λ increases, the proportion of consumers in market U increases which raises the ISP's total profit in market U. At the opposite, when λ increases, less consumers are in market R and thus the total ISP's profit in market R goes downward. These two opposite effects on the overall profit for the ISP are set by the difference between gross utilities that consumers get joining the old or the new technology, and especially on the level of, w_f^*, the gross utility that consumers get from joining the new technology. Result in Proposition 3 shows that the ISP chooses the (inner) equilibrium network investment as λ^* when w_f^* is high enough.

Let's now turn to the analysis of cross effects between the network investment from the ISP and the investment in quality of services from the CPs. More precisely, we show in the following how network investment from the ISP can be affected by an investment in content quality from the CPs. To this end, we consider that the quality for the premium content of CPi, $\overline{\theta}_i$, is strictly proportional to the quality for the basic content, θ_i. We note $\alpha_i \geq 1$ the coefficient that links both quality for CPi, then $\overline{\theta}_i = \alpha_i \theta_i$. Hereafter, this coefficient is called "the quality expansion for content". Hence, by choosing α_i, the CPi can improve the quality for its premium content. To simplify as much as possible the following analysis, we consider that CPs can increase the quality of their content when they get acces to the new technology in a same proportion, so that $\alpha_i = \alpha$, for $i = A, B$.

From Proposition 3, we get the inner equilibrium of the network investment for the ISP which can be defined as a function of α. Let's note $\lambda^*(\alpha)$, the network investment that maximizes the long-run profit of the ISP as given from Proposition 3. Hence, the following two marginal effects of the quality expansion for content on the ISP's profit can be distinguished. First, an increase of the quality expansion for content raises the total gross utility of consumers when joining the new technology, and then impacts the equilibrium access price for consumers. Doing so, an increasing value for α affects the way in which the market segmentation between both access technologies (old and new) is operated in market U, the ISP's market share in market R, and finally the overall ISP's profit. The second effect is the one that pass through the equilibrium network investment. Hence, an increasing level for the quality of premium content undoubtedly affects the ISP decision for network investment as premium contents are more valuable for consumers, and then this helps the ISP to extract more surplus from consumers in market U.

The following Proposition 4 gives the impacts of a marginal increase of the quality expansion for content.

Proposition 4 *Suppose that $w_f^* \geq \overline{w}_f$. If the quality expansion for content increases: (i) the network investment is higher; and (ii) the overall profit of the ISP increases.*

The intuition of these result is the following. Remember first that the quality expansion for content, α, increases the consumers willigness to pay for premium contents. Consequently, it raises the expected utility, EU_f^*, and finally the total gross utility, w_f^*, that consumers expected to get from the purchase of contents when they subscribe to the new technology. Doing so, the quality expansion for content increases the consumers willigness to pay in market U for the new technology and then increases the equilibrium market shares of the ISP,$1 - \widetilde{y}^*$, for the new technology. On the other hand, the quality expansion of content doesn't produce any direct effect on the equilibrium outcomes in market R. Thus, the direct effect of an increasing quality expansion for content on the overall ISP's profit is definitly positive. In addition to this direct effect, the quality expansion for content produces an indirect effect as it impacts, λ^*, the network investment decision of the ISP. Hence, as the quality expansion for content increases the consumers willigness to pay for premium, the incentive for the ISP to increase its network investment is reinforced. Indeed, an increasing λ means a larger market U, and then the possibility for the ISP to attract more consumers with the new technology and get more surplus from consumers in market U. This positive effect is larger than the negative effect that comes from a smaller size for market R, and finally a higher α increases the profitablity of network investment, $\partial \Pi_{isp}^* / \partial \lambda$, for the ISP. This is the main intuition for result (i). As both direct and indirect effect are positive, the ISP get definitly a higher overall profit from an increase of the quality expansion for content provided. This is thecase when the consumers valuation for the premium content is high enough, $w_f^* \geq \overline{w}_f$. This is the intuition for result (ii).

5 Conclusion

(To be completed)

6 References

References

[1] Bourreau, M., Cambini, C. and P. Dogan (2011), "Access Pricing, Competition, and Incentives to Migrate from "Old" to "New" Technology", Working Paper Series rwp11-029, Harvard University, John F. Kennedy School of Government.

[2] Bourreau, M., Cambini, C. and S. Hoernig (2012), "Access pricing, competition, and incentives to migrate from "old" to "new" technology", mimeo.

[3] Brito, D., Pereira, P., and J. Vareda (2010), "Incentives to invest and to give access to new technologies", mimeo.

[4] Cheng, H., Bandyopadhyay, S. and H. Guo (2011), "The Debate on Net Neutrality: A Policy Perspective", *Information Systems Research*, 22 (1).

[5] Choi, J. and B. Kim (2010), "Net Neutrality and Investment Incentives," RAND Journal of Economics, 41(3).

[6] Economides, N. and J. Tag (2009), "Net Neutrality on the Internet: A Two-sided Market Analysis", Working Paper, Stern School of Business, New York University.

[7] Economides, N. and B. Hermalin (2010), "The Economics of Network Neutrality", working paper, NET Institute, december.

[8] Hermalin, B. and M. Katz (2007), "The Economics of Product-Line Restrictions with an Application to the Network Neutrality Debate", *Information Economics and Policy*, 19, 215–248.

[9] Krämer, J. and L. Wiewiorra (2010), "Network Neutrality and Congestion-Sensitive Content Providers: Implications for Service Innovation, Broadband Investment and Regulation", Working Paper, Universit¨at Karlsruhe.

[10] Schuett, F. (2010), "Network neutrality: A survey of the economic literature", *Review of Network Economics* 9(2).

7 Appendix

7.1 Appendix Lemma 2

We note $D = (\theta_A - \theta_B)^2$ and $S = \theta_A + \theta_B$. We have:

$$EU_c^* = \frac{S}{2} + \frac{D}{4t} - \frac{t}{4} \text{ and } EU_f^* = \frac{\alpha S}{2} + \frac{\alpha^2 D}{36t} - \frac{5t}{4} + a$$

We note $\psi(\alpha) = EU_f^* - EU_c^*$, then

$$\psi(\alpha) = \frac{(\alpha-1)S}{2} + \frac{(\alpha^2-9)D}{36t} - (t-a)$$

$$\psi(\alpha) = 18t(\alpha-1)S + (\alpha^2-9)D - 36t(t-a)$$

We show that $\psi(\alpha_1) = \psi(\alpha_2) = 0$, where $\alpha_1 < 0$ and

$$\alpha_2 = \frac{3}{2D}\left(-6St + 2\sqrt{9S^2t^2 + 2tSD + D^2 + 4tD^2 - 4atD}\right)$$

We have $\psi(\infty) = \infty$, then $\psi(\alpha) \geq 0$ when $\alpha \geq \alpha_2$. We also easily show that $\alpha_2 > 1$. Hence, we have $\psi(\alpha) < 0$ when $\alpha < \alpha_2$, and $\psi(\alpha) \geq 0$ when $\alpha \geq \alpha_2$.

7.2 Appendix Lemma 3

Equilibrium price p_f^{FC} for the new technology is directly derived from the maximization problem of the ISP. (15) indicates the marginal consumer between buying the old technology and not buying in market R. Then, the first order condition $\partial\Pi_{isp}/\partial p_f = 0$ gives the best reply function for p_f that is noted $p_f(p_c)$. We define $\overline{\Pi}_{isp} = \Pi_{isp}(p_f(p_c), p_c)$ the best reply profit function. The derivative of the best reply profit function $d\overline{\Pi}_{isp}/dp_c$ at $p_c = p_c^{FC}$ is $2-\lambda-(1-\lambda)\left(w_c^* + s_c\right)/\gamma$. The derivative is non-positive if $w_c^* > \dfrac{2-\lambda}{1-\lambda}\gamma - s_c$, then the ISP has an incentive to sell the old technology to all consumers in market R.

7.3 Appendix Assumption 3.2

Let us first consider the case where market R is fully covered. From equilibrium prices p_f^{FC} and p_c^{FC}:

$$p_f^{FC} - p_c^{FC} = \frac{w_f^* - w_c^* + s_c - s_f + \gamma}{2}$$
$$p_f^{FC} + p_c^{FC} = \frac{3w_c^* + w_f^* + s_c - s_f - 3\gamma}{2}$$

Substituting p_c^{FC} and p_f^{FC} in conditions (16) and (17), we obtain:

$s_c - s_f - \gamma < w_f^* - w_c^* < s_c - s_f + 3\gamma$

$w_f^* - w_c^* > s_c - s_f - \gamma$

Then, this reduces to one condition:

$$s_c - s_f - \gamma < w_f^* - w_c^* < s_c - s_f + 3\gamma \tag{20}$$

Consider now that market R is not fully covered. From equilibrium prices:

$$p_f^* - p_c^* = \frac{w_f^* - w_c^* + s_c - s_f + \gamma}{2}$$
$$p_f^* + p_c^* = \frac{(1+\lambda)\gamma}{2(1-\lambda)} + \frac{w_f^* + w_c^* - s_c - s_f}{2}$$

Substituting p_c^* and p_f^* in conditions (16) and (17), we obtain:

$s_c - s_f - \gamma < w_f^* - w_c^* < s_c - s_f + 3\gamma$

$w_f^* + w_c^* > \dfrac{(3-\lambda)\gamma}{(1-\lambda)} - s_f - s_c$

that is:

$$s_c - s_f - \gamma < w_f^* - w_c^* < s_c - s_f + 3\gamma \tag{21}$$
$$w_f^* + w_c^* > \frac{(3-\lambda)\gamma}{(1-\lambda)} - s_f - s_c \tag{22}$$

7.4 Appendix Proposition 2

(i) The equilibrim overall ISP's profit is:

$\Pi_{isp} = \lambda\left[(p_c^* + s_c)\widetilde{y}^* + (p_f^* + s_f)(1 - \widetilde{y}^*)\right] + (1-\lambda)(p_c^* + s_c)\widehat{y}^*$

where $\widetilde{y}^* = \dfrac{w_c^* - w_f^* + s_c - s_f + 3\gamma}{4\gamma}$ $(= \widetilde{y}^{FC})$ and $\widehat{y}^* = \dfrac{(1-\lambda)(w_c^* + s_c) - \lambda\gamma}{2\gamma(1-\lambda)}$ ($\widehat{y}^*$is equal to 1 if market R is fully covered). Remark that $\dfrac{\partial p_c^*}{\partial s_f} = \dfrac{\partial p_c^{FC}}{\partial s_f} = 0$ and $\dfrac{\partial p_f^*}{\partial s_f} = \dfrac{\partial p_f^{FC}}{\partial s_f} = -\dfrac{1}{2}$

Derivative of Π_{isp} wrt s_f gives:

$$\frac{\partial \Pi_{isp}}{\partial s_f} = \lambda \left[(p_c^* - p_f^* - s_f + s_c) \frac{\partial \widetilde{y}^*}{\partial s_f} + \left(\frac{\partial p_f^*}{\partial s_f} + 1 \right) (1 - \widetilde{y}^*) \right]$$

At $s_f = s_c$:

$$\left. \frac{\partial \Pi_{isp}}{\partial s_f} \right|_{s_f = s_c} = \lambda \left[-(p_f^* - p_c^*) \frac{-1}{4\gamma} + \left(\frac{-1}{2} + 1 \right) (1 - \widetilde{y}^*) \right]$$

$$\left. \frac{\partial \Pi_{isp}}{\partial s_f} \right|_{s_f = s_c} = \lambda \left[\frac{p_f^* - p_c^*}{4\gamma} + \frac{1 - \widetilde{y}^*}{2} \right] > 0$$

This result prevails as well when market R is fully covered or not.

(ii) The consumers surplus is $SC = (1-\lambda) SC_R + \lambda SC_U$ where SC_R and SC_U are given by (8). Consider the case where market R is not fully covered, the equilibrium consumers surplus SC^* is derived from:

$$SC_R^* = (w_c^* - p_c^*) \widehat{y}^* - \frac{\gamma}{2} \widehat{y}^*$$

$$SC_U^* = (w_c^* - p_c^* - w_f^* + p_f^* + \gamma) \widetilde{y}^* - \gamma \widetilde{y}^{*2} - \frac{\gamma}{2} - p_f^* + w_f^*$$

$$\begin{aligned} \frac{\partial SC^*}{\partial s_f} &= \lambda \frac{\partial p_f^*}{\partial s_f} (\widetilde{y}^* - 1) + \lambda (w_c^* - p_c^* - w_f^* + p_f^* + \gamma - 2\gamma \widetilde{y}^*) \frac{\partial \widetilde{y}^*}{\partial s_f} \\ &= \lambda \frac{\gamma + w_f^* - w_c^* - s_c + s_f}{8\gamma} \end{aligned}$$

Then,

$$\left. \frac{\partial SC^*}{\partial s_f} \right|_{s_f = s_c} = \lambda \frac{w_f^* - w_c^* + \gamma}{8\gamma} > 0$$

(iii) Let us consider CP_i. The equilibrium overall CP_i' profit is:

$$\Pi_i^* = \left[(1-\lambda) \widehat{y}^* + \lambda \widetilde{y}^* \right] \pi_{ci} + \lambda (1 - \widetilde{y}^*) \pi_{fii}$$

$$\frac{\partial \Pi_i^*}{\partial s_f} = \lambda \frac{\partial \widetilde{y}^*}{\partial s_f} \pi_{ci} - \lambda \frac{\partial \widetilde{y}^*}{\partial s_f} \pi_{fi} - \lambda (1 - \widetilde{y}^*)$$

$$\frac{\partial \Pi_i^*}{\partial s_f} = -\lambda \frac{\partial \widetilde{y}^*}{\partial s_f} \left(\pi_{fi} - \pi_{ci} \right) - \lambda (1 - \widetilde{y}^*)$$

Then,

$$\left. \frac{\partial \Pi_i^*}{\partial s_f} \right|_{s_f = s_c} \geq 0 \text{ iff } \left(\pi_{fi} - \pi_{ci} \right) > 4\gamma (1 - \widetilde{y}^*) = w_f^* - w_c^* + \gamma$$

Finally, iff:

$$\pi_{fi}^* - \pi_{ci}^* \geq w_f^* - w_c^* + \gamma \; (> 0)$$

7.5 Appendix Lemma 5

Comparative statics on prices, market shares, CPs profits and consumer surplus:

(i) Impact of λ on prices

$\dfrac{\partial p_c^*}{\partial \lambda} = \frac{1}{2}\frac{\gamma}{(\lambda-1)^2} > 0$

$\dfrac{\partial p_f^*}{\partial \lambda} = \frac{1}{2}\frac{\gamma}{(\lambda-1)^2}$

(ii) Impact of λ on market shares

$\dfrac{\partial \widetilde{y}_c^*}{\partial \lambda} = 0$

$\dfrac{\partial \widehat{y}_c^*}{\partial \lambda} = -\frac{1}{2(\lambda-1)^2} < 0$

7.6 Appendix Lemma ??

(i) Impact of λ on the CPs profits

From Appendix Proposition **??**, we have:

$\left.\dfrac{\partial^2 \Pi_i^*}{\partial s_f \partial \lambda}\right|_{s_f=s_c} \geq 0$ if $\pi_{fi} - \pi_{ci} \geq w_f^* - w_c^* + \gamma$

Consider CP$_A$.

We assume that $\widetilde{\theta}_i = \alpha\theta_i$, $\delta_\theta = \theta_A - \theta_B$. Then $\left.\dfrac{\partial^2 \Pi_A^*}{\partial s_f \partial \lambda}\right|_{s_f=s_c} = \dfrac{\Omega_A(\alpha)}{72\gamma t}$,

where $\Omega_A(\alpha) = \delta_\theta^2\alpha^2 + 6\delta_\theta\alpha t - (9at + 9a\delta_\theta - 9t^2 + 18tw_f^* - 18tw_c^* + 18\gamma t) \geq 0$

Hence, $\left.\dfrac{\partial^2 \Pi_A^*}{\partial s_f \partial \lambda}\right|_{s_f=s_c} \geq 0$ iff $\Omega_A(\alpha) \geq 0$

We easily show that $\Omega_A(\alpha_{A1}) = \Omega_A(\alpha_{A2}) = 0$, where

$\alpha_{A1} = 3\dfrac{-t - \sqrt{a(t+\delta_\theta) + 2t(w_f^* - w_c^* + \gamma)}}{\delta_\theta} < 0$ and

$\alpha_{A2} = 3\dfrac{-t + \sqrt{a(t+\delta_\theta) + 2t(w_f^* - w_c^* + \gamma)}}{\delta_\theta}$, with $\alpha_{A1} < \alpha_{A2}$. Remark that $\Omega_A(\alpha) \longrightarrow \infty$ when α tends to ∞. Then, $\Omega_A(\alpha) \leq 0$ when $\alpha \in [\alpha_{A1}, \alpha_{A2}]$. Finally, $\Omega_A(\alpha) > 0$ when $\alpha > \alpha_{A2}$.

Consider CP$_B$.

then $\left.\dfrac{\partial^2\Pi_B^*}{\partial s_f \partial\lambda}\right|_{s_f=s_c} = \dfrac{\Omega_B(\alpha)}{72\gamma t}$,

where $\Omega_B(\alpha) = \delta_\theta^2\alpha^2 - 6\delta_\theta\alpha t - (9at - 9a\delta_\theta - 18tw_c^* + 18tw_f^* - 9t^2 + 18\gamma t) \geq 0$

Hence, $\left.\dfrac{\partial^2\Pi_B^*}{\partial s_f \partial\lambda}\right|_{s_f=s_c} \geq 0$ iff $\Omega_B(\alpha) \geq 0$

We easily show that $\Omega_B(\alpha_{B1}) = \Omega_B(\alpha_{B2}) = 0$, where $\alpha_{B1} = 3\dfrac{t - \sqrt{a(t-\delta_\theta) + 2t(w_f^* - w_c^* + \gamma)}}{\delta_\theta}$

and $\alpha_{B2} = 3\dfrac{t + \sqrt{a(t-\delta_\theta) + 2t(w_f^* - w_c^* + \gamma)}}{\delta_\theta}$, with $\alpha_{B1} < \alpha_{B2}$. Remark that $\Omega_B(\alpha) \longrightarrow \infty$ when α tends to ∞. Then, $\Omega_B(\alpha) \leq 0$ when $\alpha \in [\alpha_{B1}, \alpha_{B2}]$. For the slake of clarity, we assume that $\alpha_{B1} < 0$, that is δ_θ is significantly small. Finally, $\Omega_B(\alpha) > 0$ when $\alpha > \alpha_{B2}$.

Finally, the owerall profit of both content providers increases with network investment if $\alpha > \widetilde{\alpha} = \max\{\alpha_{B2}, \alpha_{A2}\}$.

(ii) Impact of a marginal increase of λ on the consumer surplus

We obtain the result directly from Appendix Lemma 4.

7.7 Appendix Proposition 3

(i) Investment incentives for the ISP

We define $\Pi_{isp}^*(\lambda)$ the ISP's profit as a function of λ and from (6), we obtain:

$$\Pi_{isp}^*(\lambda) = \lambda\left[(p_c^* + s_c)\widetilde{y}^* + (p_f^* + s_f)(1 - \widetilde{y}^*)\right] + (1-\lambda)(p_c^* + s_c)\widehat{y}_c^* \tag{23}$$

The long-run overall profit of the ISP is $\Pi_{isp}^*(\lambda) - c(\lambda)$, then the marginal long-run profit of the ISP is $\dfrac{\partial\Pi_{isp}^*(\lambda)}{\partial\lambda} - c'(\lambda)$. By assumption, we know that $c(\lambda)$ is convex. Let's now study how the short-run profit of the ISP is impacted with a marginal increase of λ.

From (23) and using the envelope theorem, we obtain:

$\dfrac{\partial\Pi_{isp}^*(\lambda)}{\partial\lambda} = (1-\widetilde{y}^*)(p_f^* + s_f) + (p_c^* + s_c)(\widetilde{y}^* - \widehat{y}^*) + (1-\lambda)(p_c^* + s_c)\dfrac{\partial\widehat{y}_c^*}{\partial\lambda}$, wher $\dfrac{\partial\widehat{y}_c^*}{\partial\lambda} = -\dfrac{1}{2(1-\lambda)^2}$

From (15), (14) and (18):

$\widetilde{y}^{*}-\widehat{y}^{*}=\dfrac{\gamma(3-\lambda)-(1-\lambda)(w_f^{*}+w_c^{*}+s_f+s_{c)}}{4\gamma(1-\lambda)}<0$ and $1-\widetilde{y}^{*}=\dfrac{w_f^{*}-w_c^{*}+s_f-s_c+\gamma}{4\gamma}>0$

Then,

$$\frac{\partial \Pi_{isp}^{*}(\lambda)}{\partial \lambda}=\frac{\gamma+(w_f^{*}+s_f)(1-\lambda)}{2(1-\lambda)}\frac{w_f^{*}-w_c^{*}+s_f-s_c+\gamma}{4\gamma}+$$

$$\frac{\lambda\gamma+(w_c^{*}+s_c)(1-\lambda)}{2(1-\lambda)}\left[\frac{\gamma-(w_f^{*}+w_c^{*}+s_f+s_c)}{4\gamma}\right]$$

Let's note $f(w_c^{*},w_f^{*},\lambda)=8\gamma(1-\lambda)\dfrac{\partial \Pi_{isp}^{*}(\lambda)}{\partial \lambda}$.

For the sake of clarity of the proof, we note $X=w_f^{*}+s_f$, $Y=w_c^{*}+s_c$, and $F(Y)=f(w_c^{*},w_f^{*},\lambda)$. Then, the marginal short-run profit becomes:

$$\frac{\partial \Pi_{isp}^{*}(\lambda)}{\partial \lambda}=\frac{F(Y)}{8\gamma(1-\lambda)}$$

where $F(Y)=(\gamma+X(1-\lambda))\,(X-Y+\gamma)-(\lambda\gamma+Y(1-\lambda))\,(X+Y-\gamma)$

We easily show that $F(Y_1)=f(Y_2)=0$, where $Y_1<0$ and given by:

$$Y_1=-\frac{\gamma\lambda+X(1-\lambda)+\sqrt{2X^2(1-\lambda)^2+\gamma(2X(1-\lambda)+\gamma)}}{1-\lambda}$$

$$Y_2=\frac{\gamma\lambda+X(1-\lambda)-\sqrt{2X^2(1-\lambda)^2+\gamma(2X(1-\lambda)+\gamma)}}{1-\lambda}$$

Moreover, we have $F(\infty)=-\infty\,(\lambda-1)^2<0$. Then, $F(Y)\geq 0$ when $0<Y\leq Y_2$ and $F(Y)<0$ when $Y>Y_2$.

Finally, remember here market R is not fully covered *i.e.* $Y\leq\dfrac{2-\lambda}{1-\lambda}\gamma$. Then, we need now to compare Y_2 and $\dfrac{2-\lambda}{1-\lambda}\gamma$. After some calculus, we easily show that $Y_2\geq\dfrac{2-\lambda}{1-\lambda}\gamma$ when $X\geq\dfrac{3\gamma}{1-\lambda^{*}}$. Consider this condition holds, we conclude that the short-run profit of the ISP, $\Pi_{isp}^{*}(\lambda)$, is non decreasing with λ, and the long-run profit of the ISP is concave with respect to λ. In this case, the first order condition for the maximization problem of the ISP gives the equilibrium network investment, λ^{*}, from $f(w_c^{*},w_f^{*},\lambda^{*})=8\gamma(1-\lambda^{*})c'(\lambda^{*})$, whit $w_f^{*}\geq\overline{w}_f=\dfrac{3\gamma}{1-\lambda^{*}}+s_f$.

7.8 Appendix Proposition 4

(i) We assume here that $\overline{\theta}_i = \alpha_i \theta_i$. Then, as $w_f^* = u_f + EU_f^*$ and from (13), we can defined w_f^* as a function of α. We note this function $w_f(\alpha)$. We easily show that $w_f^{'}(\alpha) = \dfrac{2\alpha(\theta_A - \theta_B)}{36t} + \dfrac{(\theta_A + \theta_B)}{2} > 0$. We define the equilibrium network investment as a function of w_f as$\lambda^*(w_f)$. Then, from the first order condition of the ISP's maximization problem, we have:

$$\frac{\partial \Pi_{isp}^*(\lambda^*(w_f), w_f)}{\partial \lambda} - c'(\lambda^*(w_f)) = 0 \tag{24}$$

Hence, differentiating (24) with respect to w_f, we obtain:

$$\left[\frac{\partial^2 \Pi_{isp}^*(\lambda^*(w_f), w_f)}{\partial \lambda^2} - c^{''}(\lambda^*(w_f))\right] \frac{\partial \lambda^*(w_f)}{\partial w_f} + \frac{\partial^2 \Pi_{isp}^*(\lambda^*(w_f), w_f)}{\partial w_f \partial \lambda} = 0$$

We assume here that the maximization problem of the ISP is concave, $\dfrac{\partial^2 \Pi_{isp}^*(\lambda^*(w_f), w_f)}{\partial \lambda^2} - c^{''}(\lambda^*(w_f)) < 0$. Then, we can deduce:

$$sgn\left(\frac{\partial \lambda^*(w_f)}{\partial w_f}\right) = sgn\left(\frac{\partial^2 \Pi_{isp}^*(\lambda^*(w_f), w_f)}{\partial w_f \partial \lambda}\right)$$

After tedious calculus, we show:

$$\frac{\partial^2 \Pi_{isp}^*(\lambda^*(w_f), w_f)}{\partial w_f \partial \lambda} = \frac{w_f + s_f - w_c - s_c + \gamma}{4\gamma} > 0 \text{ (from Assumption 3.2)}$$

We conclude that $\dfrac{\partial \lambda^*(w_f)}{\partial w_f} > 0$.

Finally, we have $\lambda(\alpha) = \lambda(w_f(\alpha))$. Then, $\lambda^{'}(\alpha) = \dfrac{\partial \lambda^*(w_f)}{\partial w_f}\, w_f^{'}(\alpha) > 0$.

(ii) The long-run profit of the ISP,$\overline{\Pi}_{isp}$, as a function of α writes $\overline{\Pi}_{isp}(\alpha) = \Pi_{isp}^*(\lambda(\alpha), \alpha) - c(\lambda(\alpha))$. Then, using the envelope theorem, we have $\dfrac{d\overline{\Pi}_{isp}(\alpha)}{d\alpha} = \left.\dfrac{\partial \Pi_{isp}^*}{\partial \alpha}\right|_{\lambda = \lambda^*(\alpha)} > 0$ (from Appendix Proposition 3)