Competition and Provision of Complementary Open Source Software

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Abstract

This paper analyzes a model in which competing firms can invest in the quality of open source software. All firms benefit from such an investment, that is, open source software is a public good. We show that, contrary to standard public goods, additional public good investments which are made by a new market entrant or by the government may lead to a higher investment of all firms, that is, a crowding-in effect. We also demonstrate that all firms may increase their production although competition becomes fiercer due to market entry.

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1 Introduction

The open source software (OSS) development has remarkably changed from a model that is driven by voluntary developers toward a serious alternative or complement to proprietary software. This has increasingly motivated firms either to contribute to OSS projects or to intensely use them. For example, in 2000, Sun revealed all source codes of StarOffice in order to found the organization OpenOffice and finally released the source code of the programming language Java under the GNU General Public License in 2006. Similarly, in the early 2000s, IBM launched the not-for-profit project Eclipse in order to promote OSS contributions and donated a source code of value US$ 40 million. At the same time, they proclaimed to support Linux across all of its servers (West (2003)).[1] By virtue of the licenses OSS is equipped with, all software developers and firms can examine, use, modify, and redistribute the freely available codes. Thus, firms like IBM, Sun, Samsung or Motorola contribute to a non-excludable public good and are not able to fully appropriate their investments in the OSS development. Even though OSS licenses are designed to mitigate opportunity costs of not commercializing contributions to OSS development by ruling out exclusive appropriation of a single firm (Gächter et al. (2009)), investments in OSS projects seem to contradict the economic rationale of free riding.

Several reasons why firms nevertheless contribute to OSS are examined in a considerable literature.[2] One reason is presented by Henkel (2006), who shows that in the case of embedded Linux, many commercial firms selectively reveal parts of their codes in order to benefit from contributions from the OSS movement. This is because firms thereby may benefit from signaling technical sophistication or from exploring new technologies which they can adapt to their needs (Osterloh and Rota (2007)).[3] There is indeed evidence that investing in the OSS movement

[1]The success of the operating system Linux is one of the most remarkable examples of OSS development; as many OSS projects, the Linux kernel is also licensed under the GNU General Public License. Today, Linux is in a wide use. It runs on embedded systems for smartphones and tablet computers such as the Google-initiated operating system Android and manufacturers like e.g., Samsung and Motorola, contribute to its development. A most recent example of a commercial firm's contribution to Linux development is Microsoft's Azure Cloud which also provides a virtual machine running Linux.

[2]Note that the incentives of firms to invest in OSS differ from those of individual programmers whose motivation is often driven by reputation, career prospects or altruism. A detailed survey of the literature on on motivation to contribute to OSS is provided by von Krogh et al. (2012).

[3]A similar idea is put forward by Kleer (2010) who sets up a game-theoretic signaling model where firms apply for government subsidies. A subsidy which is granted by the government can lead to an increase in private investments, because private investors evaluate public grants as a signal for a valuable investment.

provides firms with valuable benefits which are larger than potential losses from free riding. For example, Stam (2009) identifies conditions under which the contribution of software firms in the OSS movement improves their financial and innovative performance. The analysis shows that firms receive benefits from making contributions to the OSS movement which are of decreasing marginal returns indicating that there exist an optimal level of contribution to the OSS community. In a simple game-theoretic two-firm model, Harhoff et al. (2003) demonstrate that, depending on parameters like the degree of competition between the two firms and the specialization of their innovation, software firms might benefit from revealing and diffusing their source code. A further reason is that firms set-up OSS projects in order to foreclose the development of a standard around a technology licensed by a competitor. As documented by West (2003), in the 1990s, IBM tried to respond to Microsoft's dominance by trying to win allies for their own standards also by leading open source organizations to support their technology initiatives.

The most prominent reason why firms contribute to OSS is that they offer expertise in a commercial market segment that is complementary to OSS.[4] Firms have a hybrid business model which combines the supply of proprietary and OS software (see e.g. Bonaccorsi et al. (2006)). As pointed out by e.g. Lerner and Tirole (2002, 2005), contributing to OSS development and thereby improving the quality of OSS bolsters the demand for proprietary products and services in complementary market segments.[5] Even though the companies cannot directly capture the value of a quality improvement, they can profit indirectly by selling more complementary proprietary goods at a potentially higher price.

However, as also emphasized by Lerner and Tirole (2002, 2005), OSS shares an important property of a public good, which is, that additional contribution to it leads to a crowding-out effect, that is, existing firms contribute less due to the well-known free-riding problem.[6] Clearly, this has important policy implications, i.e., governmental spending on OSS or fostering entry of new firms that also contribute to OSS leads to a reduction in the contribution of existing firms

[4]Contribution to OSS development can not only be done by subsidizing projects or by revealing source codes and thus initiating an OSS project but also by strategically sponsoring and employing individuals who work in a firm's interest in an OSS community (Dahlander and Wallin (2006)).

[5]For example, IBM gained strategic advantages from Linux by offering specialized services to large corporate customers (West(2003)).

[6]See e.g. Bergstrom et al. (1985) for a detailed analysis.

and should thereby taken with caution.[7] However, recent empirical literature caused doubts on the crowding-out hypothesis. For example, Gonzáles and Pazó (2008) found that, in the case of Spanish manufacturing firms, no crowding-out effect occurs. Similarly, Aerts and Schmidt (2008) who use data from Flamish and German Community Innovation Survey (CIS), show that publicly funded firms are significantly more active in R&D than firms which did not received public R&D funding, and reject the crowding-out hypothesis at a robust level. These findings cause problems for well-guided policy making because they suggest that the usual modeling assumptions on which these policy implications are based do not necessarily hold for OSS.

The aim of this paper is to explore the question of crowding-out in more detail. We first set-up a very general model in which firms produce a private good and can invest into the quality of a public good, that is contribution to OSS. Firms are (imperfect) competitors in the proprietary market segment, and the private good is complementary to the public good. In particular, increased investments in the quality of the public good makes the private good more valuable, implying an outward shift of the demand curve. All firms benefit from that increase in investment confirming that OSS shares the standard properties of a public good.

In this environment we first show that the quantity of the private good and the provision of the public good are too small from a social perspective. This result is standard in the literature on imperfect competition and public good provision. In order to analyze the effects of policy intervention on the market outcome, we investigate government contribution to OSS. Such contributions are widespread and range e.g., from the adoption of OSS technologies in governmental agencies to direct subsidies for OSS projects.[8] We show that under a wide array of circumstances such an increase in government spending does not lead to a crowding-out of private contribution but in fact to a crowding-in. The reason for this result is that firms benefit from the increased quality of OSS through an increased demand for their product in the proprietary market segment, i.e., they receive larger revenues and therefore raise the produced quantity of the private good. This affects their incentive to contribute to OSS because such investments improve their performance in the proprietary market segment. In particular, since firms sell more

[7] Schmidt and Schnitzer (2002) also argue that government subsidies for OSS projects should be limited to basic research.

[8] A detailed survey of policies towards OSS projects is provided by the Center for Strategic and International Studies (2012).

of the private goods, their incentive to enhance the quality of the public good increases as well. We demonstrate that this effect is generally very strong and leads to a crowding-in of private investments. For example, independent of the cost functions, if demand for the proprietary good as linear, a crowding-in effect occur.

In a next step, we analyze whether the policy instrument of fostering market entry leads to a crowding-in or a crowding-out of firms' contribution to OSS.[9] As free-riding can be assumed to be stronger when the number of market entrants increases, one would rather expect a lowered incentive of firms to invest in OSS technology. By contrast, we demonstrate that a similar effect as in case of direct government contribution can be observed in our setup. An entering firm, although being a competitor in the proprietary market to incumbent firms, also contributes to the quality of OSS which in turn leads to an enhancement of the market value of the incumbents' private products. Therefore, a crowding-in effect might occur because incumbents are then more incentivized to increase investments in OSS. Moreover, this effect can be large so that incumbents even increase their production of the private good after the market entry of a new firm. This result runs counter to standard intuition in oligopoly models. However, when OSS is considered to be complementary to the proprietary market, a new entrant might enhance consumers' valuation for the private products through its investment in OSS so that this results in a large outward shift of the demand curve. Consequently, incumbent firms are then induced to broaden production in the proprietary market. Thus, our analysis suggests that the fear of the free-riding problem in OSS is not well justified and often crowding-in can occur as a result of an increased number of firms contributing to OSS. We can therefore formulate the policy implication that stimulating market entry by the government spurs investments in OSS technologies to larger extent than existing models suggest.

We finally compare the two policy instruments with regard to their efficiency in crowding-in properties. We find that if firms' equilibrium investment into the quality of OSS technology is relatively small, direct government contribution leads to a larger crowding-in effect. The same result holds if the demand in the proprietary good market is very elastic. Since the

[9]For example, by using data on German ICT firms, Fritsch and von Engelhardt (2010) show that entry barriers are often low for OSS firms. Moreover, entry is stimulated by governments, e.g., by funding or providing grants to start-up companies. For example, in 2005, IBM and the Israeli Ministry of Industry, Trade and Labor made an agreement that offers grants of up to $100,000 for open source start-ups (see: Center for Strategic and International Studies (2012)).

market structure and the elasticity are usually relatively easy to observe, our analysis gives some guidance on which policy instrument should be the preferred one, given that the reactions of incumbents are a major issue.

The theoretical literature on incentives to contribute to OSS is relatively scarce. Johnson (2002) develops a model with individual user-programmers who obtain benefits from contributing to an OSS project. He shows that the probability that the project will be made does not increase with the number of developers. But as mentioned earlier, incentives of firms to invest in OSS differ from those of individual programmers. In our model we focus on firms' motivation to invest in OSS development when they have a hybrid business model which means that they compete at the same time in a market segment for a proprietary good that is complementary to OSS. The paper closest to ours is von Engelhardt and Maurer (2010) who also examine profit-maximizing firms in their model. They consider a two-stage model in which firms first decide to invest in OSS and then compete in a proprietary market. They allow firms to produce differentiated proprietary products but restrict their analysis to linear demand functions. By contrast, we provide a general setup which is modeled as one-shot game. This reflects the idea, that competition and innovation in OSS technology often occur constantly and at the same time.[10]

The rest of the paper is organized as follows: In the next section we set out the model and solve for the equilibrium. In Section 3 we analyze the impact of government contribution to the public good on the market outcome. In Section 4 we discuss if a new market entrant may lead to a higher production of the private good and to a higher level of investments in the public good. We compare this result with our findings from Section 3. In Section 5 we conclude. All proofs are relegated to the Appendix.

[10]Our paper is also closely related to the theoretical literature on R&D spillovers in oligopoly. D'Aspremont and Jacquemin (1988) and Suzumura (1992) develop two-stage models in which firms may decide cooperatively about cost-reducing R&D in the first stage but engage in quantity competition in the second stage. In contrast to these models on R&D spillovers, we consider firms which do not invest cooperatively in the public good and we assume that spillovers affect the demand function instead of the cost function.

2 Model and Equilibrium

Consider an industry with n firms indexed by $i = 1, ..., n$. Each firm i has two choice variables. It produces a quantity x_i of a homogeneous private good and makes an investment of y_i in the homogeneous non-excludable public good. The quality of the public good is additively modeled as $Y = \sum_{i=1}^{n} y_i$ and the aggregate quantity of the private good is $X = \sum_{i=1}^{n} x_i$. This let us define $X - x_i = X_{-i}$ and $Y - y_i = Y_{-i}$. Firms simultaneously set their quantities of the private good and their investment in the public good.

The inverse demand function for the private good is given by $p = p(X, Y)$, which we assume to be twice continuously differentiable. We denote the partial derivative of $p(X, Y)$ with respect to the first argument, the aggregate quantity, by $p_1(X, Y)$, and the partial derivative with respect to the second argument, the quality of the public good, by $p_2(X, Y)$. The consumers' willingness to pay decreases in the quantity of the private good, that is $p_1(X, Y) < 0$, and increases in quality of the public good, $p_2(X, Y) > 0$. The latter implies that the public good and the private good are complements. Additionally, we impose concavity of the demand function in an increase in the quality of the public good and in an increase in the quantity of the private good, that is $p_{22}(X, Y) \leq 0$ and $p_{11}(X, Y) \leq 0$. The rise in the willingness to pay from an increase in quality is larger for the first units, i.e., $p_{12}(X, Y) \leq 0$. Both assumptions are natural for many circumstances.

To illustrate this setup in terms of information technology, consider a proprietary specific service (the private good) which is complementary to open source software (the public good). Firms might now invest in the public good, although such contribution to open source technology is not directly appropriable. They benefit indirectly from an increased consumers' willingness to pay for the private good due to the quality improvement of the open source environment.

Firm i faces the costs of $c(x_i)$ for the production of the private good and incurs costs of $k(y_i)$ to increase the quality of the public good by y_i. We suppose that both cost functions are strictly increasing and convex, and that fixed costs are equal to zero. Writing $p(X, Y)$ as $p(x_i + X_{-i}, y_i + Y_{-i})$, the profit of firm i can then be formulated as

$$\pi_i = x_i p(x_i + X_{-i}, y_i + Y_{-i}) - c(x_i) - k(y_i). \tag{1}$$

To ensure strict concavity of the profit function, we assume that $2p_1(X, Y) + x_i p_{11}(X, Y) -$

$c''(x_i) < 0$ and that $p_2(X, Y)$ is small relative to $c''(x_i)$ and $k''(y_i)$.[11] The first assumption is standard in models with Cournot competition, while the second guarantees that setting y_i to infinity can never be optimal.

We can write the first-order conditions of (1) as

$$\frac{\partial \pi_i}{\partial x_i} = p(X, Y) + x_i p_1(X, Y) - c'(x_i) = 0 \tag{2}$$

and

$$\frac{\partial \pi_i}{\partial y_i} = x_i p_2(X, Y) - k'(y_i) = 0. \tag{3}$$

Equation (2) is the standard optimality condition for the private good which equates marginal costs and marginal revenue. Equation (3) explains the indirect effect of an investment of firm i in the public good on its profit. A marginal increase in quality of the public good rises the consumers' willingness to pay for the private good and so the revenue of firm i. This must be equated with the marginal costs of an increase in quality. Given our assumptions we can now state the equilibrium of the game:

Lemma 1. *There exists a unique symmetric Nash-Equilibrium, in which all firms produce a quantity x^* of the private good and invest y^* in the public good, where x^* and y^* are implicitly defined by*

$$p(X^*, Y^*) + x^* p_1(X^*, Y^*) - c'(x^*) = 0 \quad \textit{and} \quad x^* p_2(X^*, Y^*) - k'(y^*) = 0,$$

with $X^* = nx^*$ and $Y^* = ny^*$.

The equilibrium uniqueness allows us to perform comparative-static analysis with respect to government contribution and market entry. Before turning to these analyses, we can briefly compare the equilibrium quantity x^* and the quality investment y^* with the socially optimal

[11] A derivation of these assumptions can be found in the Appendix.

one. Welfare in our model is given by

$$W(X,Y) = \int_0^X p(a,Y)\mathrm{d}a - \sum_{i=1}^{n} \left[c(x_i) + k(y_i)\right]. \tag{4}$$

Differentiating with respect to x_i and y_i let us determine the socially optimal values x^W and y^W which are implicitly given by $p(X^W, Y^W) - c'(x^W) = 0$ and $\int_0^{X^W} p_2(a, Y^W)\mathrm{d}a - k'(y^W) = 0$, with $X^W = nx^W$ and $Y^W = ny^W$. A comparison between the market outcome with the socially efficient outcome let us obtain the following intuitive lemma.

Lemma 2. *The market provision of quantity and quality is too low compared to the socially efficient level, that is, $X^W > X^*$ and $Y^W > Y^*$.*

These results are intuitive. Since there is imperfect competition in the commercial market segment, firms obtain profits implying that the price exceeds marginal costs. Thus, it is apparent that the aggregate quantity is too small compared to the optimal outcome. Regarding the optimal quality of the public good, the effect driving a wedge between the socially optimal quality and the equilibrium one is of twofold nature. First, a firm obtains a single price for each unit of the private good it sells. This price represents the valuation of the marginal consumer. Hence, its incentive to invest in quality is catered to the valuation increase of the marginal consumer. But consider that all consumer above the marginal one benefit to a larger extent from the quality increase. This is taken into account by the social planner but not by the private firm.[12] Second, since the market is comprised of an oligopoly, all firms produce positive quantities, implying that the price of the private good is lower than in duopoly. This also reduces the incentive for firm i to invest in quality.

3 Government Contribution

In this section, we investigate a situation of quality improvement of the public good by government intervention. Suppose that the government contributes to the quality of the public

[12]See Spence (1975) for the seminal paper on this effect.

good, e.g., by funding research projects. We denote this government investment by y_G and assume that it additively contributes to the quality of the public good, as is the case with quality improvements of private firms. As pointed out in the Introduction, our main interest is whether government investments cause a crowding out of private investments in the public good as predicted by the standard public good literature. In those models, firms (or households) invest in the public good until marginal revenue (or marginal utility) equals marginal costs. If the government increases investments in the public good, marginal revenue (or utility) falls which implies that firms (or households) invest less. Therefore, government investments crowd out private investments. As the next Proposition shows, in our set-up this crowding out effect does not necessarily occur. In contrast, for a large class of cases, crowding-in of private investments due to government contribution occurs.

Proposition 1. *Given that the government increases the quality of the public good by $y_G > 0$, each firm invests more in the quality of the public good if and only if $p_2 > \sqrt{p_{22}x^*(2p_1 + p_{11}x^* - c'')} - x^*p_{12}$.*

The Proposition shows that if p_2 is large enough, a crowding-in effect occurs. The intuition behind this result is the following: If the government invests into the public good, the quality increases which implies that the marginal revenue from an additional quality increase falls. This induces each firm to lower its investment in the quality of the public good. This is the standard effect which is well known from the public good literature. However, in our case, a second effect has to be considered which stems from the private good market. A higher quality of the public good increases the consumers' willingness-to-pay for the private good which leads to higher profits of each firm per unit of output. Therefore, each firms optimally produces a higher quantity. The quantity increase in turn incentivizes each firm to invest in the quality of the public good. Thus, a quality investment pays off to a larger extent and if this countervailing effect is strong enough, firms increase their investments. In particular, if the increase in the willingness-to-pay from an additional investment is very high, which holds in case p_2 is very high, the government's investment spurs further investments of the private firms because then the quantity increase is of a large extent.

We therefore obtain that through the effect which stems from the private good market, the government investment and the private investment in the public good can be strategic complements. This can never occur if firms would not benefit from the public good's quality, which affects consumers' valuation for the complementary private good. To be more precise, suppose that the quantity of each firm, x_i, is fixed, that is, a change in the quality of the public good does not affect a firm's production of the private good. Hence, there would always be crowding-out of private investments in the public good, i.e., an increase in y_G leads to a decrease in y^*. Government investment and private investment would therefore be strategic substitutes.

Let us now turn to the analysis of the effect how y_G affects the quantity of the private good which is produced by the oligopoly.

Proposition 2. *Given that the government increases the quality of the public good by $y_G > 0$, each firm produces more of the private good if and only if $p_2 > -x^* p_{12}$.*

The Proposition shows that if the increase in consumers' willingness-to-pay for the private good through an improvement of the quality of the public good is large, each firm increases its production. This effect is intuitive since a right-shift in the demand curve implies that the marginal revenue of the firms increases. Thus, each firm optimally reacts by producing a higher quantity and thereby, lowering marginal revenue and increasing marginal costs.

The Proposition also shows that this natural effects can be reversed if p_{12} is highly negative. In this case, consumers with a high willingness-to-pay benefit a lot from a quality extension while those with a low willingness-to-pay benefit very little. This implies that the demand curve becomes less elastic. Therefore, firms benefit from reducing their quantities and exploiting the consumers with high valuations to larger extent.

A prominent demand function, due to its simplicity, is the linear one, in which case $p_{11} = p_{22} = p_{12} = 0$. As is evident from the formulas in the two propositions above, in the linear case we obtain a clear-cut result on the reaction of private firms as response to an increased government contribution to the public good.

Corollary 1. *Under a linear demand function, an increase in the quality of the public good by $y_G > 0$ induces each firm to invest more in the quality of the public good as well as to produce a larger quantity of the private good.*

Hence, with a linear demand function we always obtain a crowding-in effect. It is remarkable to note that this result is independent of the cost functions for the private and the public good. Clearly, it also holds if the demand function has some curvature which does not depart too much from linearity. The result shows that the standard prediction for private contributions to public goods does not hold if firms contribute to open source software which is a complement to their proprietary software. Governmental contribution to open source software is likely to stimulate further contribution by private firms and also leads to an increase in output.

4 Market Entry

In this section we investigate the effects of market entry on the private good production and the level of investment for the public good. Johnson (2002) argues that, in case of individual programmers, the effect of an increased number of contributors on the incentive of each programmer to contribute to an OSS project is ambiguous. On the one hand, incentive to free-ride is amplified while on the other hand, redundant contribution is curbed. When focusing on software firms which are active in OSS development as well as in a proprietary market segment, fostering market entry, e.g., via subsidies, can be a helpful instrument from a policy perspective to improve market outcomes for both, the private and the public good. However, due to fiercer competition in the proprietary market, one would expect that in equilibrium all firms lower their production for the private good as well as their investments in the quality of the public good, e.g., a crowding-out effect to occur. As the next proposition shows, this effect does not necessarily dominate.

Proposition 3. *If the number of firms increases, each firm invests a larger amount in the quality of the public good and produces a larger quantity of the private good if* p_2 *is large enough.*

The intuition behind the result on the public good is similar to the one of the last section. In the last section, the government invested in public good quality, while now a new market entrant increases the level of investments. This may stimulate the quantity production of the incumbent firms and thereby also induces them to invest more. It is interesting that this effect also occurs with a new entrant who, in contrast to the government, also produces a positive quantity of the private good, which due to a lower market price, also reduces the value of private good production. However, as Proposition 3 also shows, entry can even lead to an increase in the quantity which is produced by the incumbent firms. This result perhaps seems surprising and is at odds with conventional results from Cournot oligopoly. The intuition behind is the following: If a new firm enters the market, it increases the produced quantity of the private good and the level of the quality of the public good. The first effect imposes a negative impact on consumers' willingness to pay for the private good, while the second effect imposes a positive impact on consumers' valuation for the private good. If p_2 is large, the second effect dominates and the marginal revenue from the private good increases inducing each firm to optimally produce a larger quantity. Therefore, firms' quantities are strategic substitutes if we solely consider the private good market. But the produced quantities become strategic complements if we additionally consider a public good investment, given that this investment is sufficiently important. Thus, the positive effect on firms' incentive to invest in the public good which stems from the private good market might also overcome negative effects evolving from free-riding.

As in the last Section we consider an example with linear demand. This yields the following result:

Corollary 2. *If the demand function is linear, then an increase in the number of firms leads to increase in x^* and y^* if and only if $p_2 > -(p_1 x^*)/y^*$.*

In contrast to the case of governmental contribution, the result with respect to market entry is less clear. In case of a linear demand function, we found that an increase in government contribution to the public good always leads to a crowding-in effect, this is no longer necessarily the case for market entry. The reason is that a newly entering firm also expands the quantity of the private good which has a negative impact on its price. Hence, incumbents are more likely to reduce their quantities and therefore also their investment in the public good.

We can finally compare the effects on the incumbents' reactions from the two policy instruments, government contribution and fostering market entry. This is of importance for policy makers when contemplating these two policies to promote OSS, taking the responses of previous firms into account. As we have already shown, the fear of crowding-out effects is of much less importance than standard models would predict, and crowding-in effects are likely to arise. The following lemma shows under which conditions these crowding-in effects are larger given one or the other regime.[13]

Proposition 4. *Consider the case with a linear demand function. The crowding-in effect for y_i^* and x_i^* under government contribution is larger than under market entry if and only if $p_2 > p_1 x^*/|1 - y^*|$.*

The result shows that when y^* is relatively small, i.e., below 1, then the right-hand-side of the formula in the proposition is negative, and the crowding-in effect of government contribution is always larger than in case of market entry. Even if y^* is above 1, the same result holds as p_2 is large relative to p_1. This shows that in a market with many firms, where each one contributes only a small amount to the public good (y^* small) or in a market where the demand function is relatively elastic ($|p_1|$ small), government investment in the quality of the public good leads to a larger crowding-in effect than market entry. Again, it is important to consider that these results do not depend on the cost functions and hold for both, the public and the private good. In addition, the relative investments of firms in OSS and the price elasticity of demand are variables that are usually easy to measure, implying that our results give some guidance for policy makers. Therefore, our result give not only clear predictions under which conditions crowding-in effects are likely to arise, they are also helpful for public policy which market intervention to pursue.

5 Concluding Remarks

This paper analyzed a model in which firms compete in a commercial market and invest in a complementary public good like open source software. We have shown that there is not necessarily a crowding-out effect of private investments in the public good due to an exogenous change in the investment level. For example, governmental contribution or additional investments of a new market entrant lead to further investments in the public good quality of each commercial firm under fairly large conditions. This occurs because an increase in the quality of the public good increases consumers' willingness-to-pay and therefore, each firms has an incentive to broaden

[13] To be able to obtain results that are easily interpretable, we restrict our attention to the case of linear demand. However, the main insights also apply for general demand.

its private good production. This in turn leads to higher investment incentives of each firm. We also compare the two policy instruments of government contribution and fostering market expansion with respect to their responses of incumbent firms. Here we obtain the result that if incumbents' equilibrium investments are not very large, government intervention leads to a higher crowding-in effect.

We conducted our analysis in a Cournot setup, that is, each firm sets its quantity in the private good market and quantities are strategic substitutes. Our results are, however, not restricted to this set-up, and should even be amplified under (differentiated) Bertrand competition. The reason is that in this case, an increase in the willingness-to-pay induces each firm to set a higher price which, ceteris paribus, raises the revenues of all other firms. Thus, each firm has an additional incentive to invest in the quality of the public good. By contrast, in our model, a higher willingness-to-pay results in a higher produced quantity of each firm which is to the detriment of the rivals. Nevertheless, we found a crowding-in effect in the private good production. Thus, our mode of competition in the private good market weakens this effect and our results are likely to get stronger because strategy variables in the private good market are strategic complements.

A Appendix

Proof of Concavity Assumptions

The Hessian matrix of the maximization problem is

$$H(\pi_i) = \begin{bmatrix} 2p_1 + x_i p_{11} - c'' & p_2 + x_i p_{12} \\ p_2 + x_i p_{12} & x_i p_{22} - k'' \end{bmatrix},$$

where for notational convenience we dropped the arguments of the functions. In order to ensure concavity, the Hessian matrix has to be negative definite. The Hessian is negative definite if the two first principal leading minors are negative and if the determinant is positive which is ensured by the following conditions: $2p_1+x_ip_{11}-c'' < 0$, $x_ip_{22}-k'' < 0$, and $(2p_1+x_ip_{11}-c'')(x_ip_{22}-k'') > (p_2 + x_ip_{12})^2$. It is readily checked that our assumptions guarantee that all these conditions are fulfilled.

Proof of Lemma 1.

We start by showing that any Nash equilibrium must be a symmetric one, that is, $x_1^* = ... = x_n^* = x^*$ and $y_1^* = ... = y_n^* = y^*$. Suppose to the contrary that $x_1^* > x_2^*$. We then have

$$\frac{\partial \pi_2}{\partial x_2} - \frac{\partial \pi_1}{\partial x_1} = (x_2^* - x_1^*)\, p_1(X^*, Y^*) + c'(x_1^*) - c'(x_2^*),$$

with $X^* = \sum_{i=1}^n x_i^*$ and $Y^* = \sum_{i=1}^n y_i^*$. At the optimal value for x_1, we must have $\partial \pi_1 / \partial x_1 = 0$. Since $p_1(X^*, Y^*) < 0$ and the marginal costs are increasing, we have

$$\frac{\partial \pi_2}{\partial x_2} = (x_2^* - x_1^*)\, p_1(X^*, Y^*) + c'(x_1^*) - c'(x_2^*) > 0,$$

which is a contradiction. Hence, x_1^* must be equal to x_2^*. A similar proof can be applied to show that symmetric investments will occur in a Nash-Equilibrium.

We now prove uniqueness of the equilibrium. Suppose that (X^*, Y^*) is an equilibrium. It then must have

$$p(X^*, Y^*) + \frac{X^*}{n} p_1(X^*, Y^*) - c'(X^*/n) = 0 \tag{A.5}$$

and

$$\frac{X^*}{n} p_2(X^*, Y^*) - k'(Y^*/n) = 0. \tag{A.6}$$

To simplify the notation, let us denote the left-hand sides of $(A.5)$ and $(A.6)$ by $f(X^*, Y^*)$ and $g(X^*, Y^*)$, respectively. Differentiating these expressions with respect to X^* and Y^* we obtain,

$$f_{X^*}(X^*, Y^*) = p_1(X^*, Y^*) \left(1 + \frac{1}{n}\right) + \frac{X^*}{n} p_{11}(X^*, Y^*) - \frac{c''(X^*/n)}{n},$$

$$f_{Y^*}(X^*, Y^*) = p_2(X^*, Y^*) + p_{12}(X^*, Y^*) \frac{X^*}{n},$$

$$g_{X^*}(X^*, Y^*) = \frac{p_2(X^*, Y^*)}{n} + p_{12}(X^*, Y^*) \frac{X^*}{n} \quad \text{and} \quad g_{Y^*}(X^*, Y^*) = p_{22}(X^*, Y^*) \frac{X^*}{n} - \frac{k''(Y^*/n)}{n}.$$

We can now employ the mean value theorem. We denote $(X', Y') = (X^* + \Delta X, Y^* + \Delta Y)$. By the mean value theorem, there exist λ and μ, with $0 < \lambda, \mu < 1$ such that

$$f(X', Y') = f_{X^*}\left((X^*, Y^*) + \lambda(\Delta X, \Delta Y)\right) \Delta X + f_{Y^*}\left((X^*, Y^*) + \lambda(\Delta X, \Delta Y)\right) \Delta Y$$

and

$$g(X^{'},Y^{'}) = g_{X^*}\left((X^*,Y^*)+\mu(\Delta X,\Delta Y)\right)\Delta X + g_{Y^*}\left((X^*,Y^*)+\mu(\Delta X,\Delta Y)\right)\Delta Y,$$

where we used the fact that $f(X^*,Y^*) = g(X^*,Y^*) = 0$. Now we look for $(X^{'},Y^{'})$ that satisfy $f(X^{'},Y^{'}) = g(X^{'},Y^{'}) = 0$. We then must have (omitting the arguments)

$$\begin{aligned} f_{X^*}\Delta X + f_{Y^*}\Delta Y &= 0, \\ g_{X^*}\Delta X + g_{Y^*}\Delta Y &= 0. \end{aligned}$$

From our assumption that $p_2(X,Y)$ is small relative to $c''(X/n)$ and $k''(Y/n)$, it follows that $|f_{X^*}| > |f_{Y^*}|$ and $|g_{Y^*}| > |g_{X^*}|$. But this implies that the determinant of the system above, $f_{X^*}g_{Y^*} - f_{Y^*}g_{X^*}$ is positive. Hence the system has a unique solution, which is given by $\Delta X = \Delta Y = 0$. As a consequence, (X^*,Y^*) are the only values satisfying $f(X,Y) = g(X,Y) = 0$. ■

Proof of Lemma 2.

We start with a comparison of X^W and X^*. Suppose that $Y^W > Y^*$. Then, one can directly obtain from the first-order conditions for X^W and X^* that the first-order condition for X^* is negative at X^W since $p_1(X,Y) < 0$. It follows that $X^W > X^*$.
To prove that $Y^W > Y^*$, we use the two first-order conditions for Y^W and Y^*. The one for Y^W is given by

$$\int_0^{X^W} p_2(a,Y^W)\mathrm{d}a - k'(y^W) = 0, \tag{A.7}$$

while the one for Y^* is given by (A.6) which is equivalent to

$$\frac{1}{n}\int_0^{X^*} p_2(X^*,Y^*)\mathrm{d}a - k'(Y^*) = 0. \tag{A.8}$$

Suppose now by contradiction that $Y^W = Y^*$. We can then identify several reasons why (A.7) exceeds (A.8). First, $n > 1$. Second, if $X^W > X^*$, we have that $p_2(a,Y^W) > p_2(X^*,Y^*)$ since $p_{12}(X,Y) \leq 0$. Third, if $X^W > X^*$, the area covered by the integral in (A.7) is larger than that of (A.8). Thus, $Y^W > Y^*$. ■

Proof for Propositions of Section 3

Suppose that the government contributes y_G to the quality of the public good. The Nash-Equilibrium is then determined by the two first-order conditions $p(X^*, Y^*+y_G)+x^*p_1(X^*, Y^*+y_G) - c'(x^*) = 0$ and $x^*p_2(X^*, Y^*+y_G) - k'(y^*) = 0$. In order to analyze how x_i^* and y_i^* change with y_G, let us rewrite these conditions to

$$p(x_i^* + X_{-i}^*, y_i^* + Y_{-i}^* + y_G) + x_i^* p_1(x_i^* + X_{-i}^*, y_i^* + Y_{-i}^* + y_G) - c'(x_i^*) = 0$$

and

$$x_i^* p_2(x_i^* + X_{-i}^*, y_i^* + Y_{-i}^* + y_G) - k'(y_i^*) = 0.$$

Let us now apply comparative statics with respect to y_G. Totally differentiating both conditions (by virtue of the symmetry of x, we can omit firm indexes and arguments) yields[14]

$$\begin{aligned} \left(2p_1 + x^*p_{11} - c''\right) dx_i^* + \left(p_2 + x^*p_{12}\right) dy_i^* &= -\left(p_2 + x^*p_{12}\right) dy_G \\ \left(p_2 + x^*p_{12}\right) dx_i^* + \left(x^*p_{22} - k''\right) dy_i^* &= -\left(x^*p_{22}\right) dy_G. \end{aligned} \tag{A.9}$$

Using *Cramer's rule* allows us to compute the solution to the system (A.9) which can be written as

$$\frac{dy_i^*}{dy_G} = \frac{p_2\left(p_2 + 2x^*p_{12}\right) - p_{22}x^*\left(2p_1 + x^*p_{11} - c''\right) + p_{12}^2(x^*)^2}{\Delta}$$

and

$$\frac{dx_i^*}{dy_G} = \frac{k''(p_2 + x^*p_{12})}{\Delta},$$

with $\Delta = k''(c'' - 2p_1 - x^*p_{11}) - c''x^*p_{22} + p_{22}x^*(2p_1 + xp_{11}) - p_2(p_2 + 2p_{12}x^*) - 2p_{12}^2(x^*)^2$. The determinant of the coefficient matrix Δ is positive by our assumption that k'' and c'' are large relative to p_2. It follows that $dy_i^*/dy_G > 0$ if and only if $p_2 > \sqrt{p_{22}x^*(2p_1 + p_{11}x^*p_{11} - c'')} - x^*p_{12}$ and $dx_i^*/dy_G > 0$ if and only if $p_2 > x^*p_{12}$. Thus, if p_2 is sufficiently large, both dy_i^*/dy_G and dx_i^*/dy_G are positive. In addition, if the demand function is linear, that is, $p_{11} = p_{22} = p_{12} = 0$, we have that dy_i^*/dy_G and dx_i^*/dy_G are surely positive since $p_2 > 0$. ■

[14] A general compendium on comparative statics for oligopolies is provided by Dixit (1986).

Proof for Propositions of Section 4

Rewriting the first-order conditions that implicitly determine x_i^* and y_i^* to include n, we obtain

$$p\left((n-1)x^* + x_i^*, (n-1)y^* + y_i^*\right) + x_i^* p_1\left((n-1)x^* + x_i^*, (n-1)y^* + y_i^*\right) - c'(x_i^*) = 0$$

and

$$x_i^* p_2\left((n-1)x^* + x_i^*, (n-1)y^* + y_i^*\right) - k'(y_i^*) = 0.$$

In order to apply comparative statics let us again totally differentiate both conditions. Omitting the arguments for simplicity allows us to write

$$\begin{aligned} \left(2p_1 + x^* p_{11} - c''\right) dx_i^* + \left(p_2 + x^* p_{12}\right) dy_i^* &= -\left(x^* p_1 + y^* p_2 + x^{*2} p_{11} + x^* y^* p_{12}\right) dn \\ \left(p_2 + x^* p_{12}\right) dx_i^* + \left(x^* p_{22} - k''\right) dy_i^* &= -\left(x^{*2} p_{12} + x^* y^* p_{22}\right) dn. \end{aligned} \tag{A.10}$$

Again, we use *Cramer's rule* and solve (A.10) for dy_i^*/dn and dx_i^*/dn in the same way as above. Thus, we have

$$\frac{dy_i^*}{dn} = \frac{p_2^2 y^* + p_2 x^* \left(p_{11} x^* + p_1 + 2p_{12} y^*\right) + p_{22} x^* y^* (c'' - 2p_1 - x^* p_{11}) + x^{*2} p_{12}(c'' + y^* p_{12} - p_1)}{\Delta}$$

and

$$\frac{dx_i^*}{dn} = \frac{p_2\left(k'' y^* + p_{12}(x^*)^2\right) + k'' x^* \left(p_1 + x^* p_{11} + y^* p_{12}\right) - (x^*)^2 \left(p_1 p_{22} + x^* p_{11} p_{22} - x^* p_{12}^2\right)}{\Delta}.$$

Again, it can be shown that dy_i^*/dn and dx_i^*/dn are positive if p_2 is sufficiently large. The linear case clarifies this result. Suppose now a linear demand function, that is $p_{11} = p_{22} = p_{12} = 0$. This allows us to rewrite both equations as

$$\frac{dy_i^*}{dn} = \frac{p_2^2 y^* + p_2 x^* p_1}{\Delta}$$

and

$$\frac{dx_i^*}{dn} = \frac{p_2 k'' y^* + k'' x^* p_1}{\Delta}.$$

It is then easy to see that both dy_i^*/dn and dx_i^*/dn are positive if $p_2 > -(p_1 x^*)/y^*$.

We can now compare dy_i^*/dn and dx_i^*/dn that we obtained in Section 4 with dy_i^*/dy_G and dx_i^*/dy_G obtained in Section 3. To obtain results that are relatively easy to interpret we constrain our attention to the case with linear demand function. Doing so yields that

$$\text{sign}\left\{\frac{dy_i^*}{dy_G} - \frac{dy_i^*}{dn}\right\} = \text{sign}\left\{\frac{dx_i^*}{dy_G} - \frac{dx_i^*}{dn}\right\} = \text{sign}\left\{p_2(1-y^*) - p_1 x^*\right\}.$$

Therefore, if $y^* < 1$ or $p_2 > p_1 x^*/(1-y^*)$, the sign is positive. ■

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